

# The physics of networks. Analyzing structural properties of the EU-funded R&D projects network by computer simulations

Vienna/Seibersdorf 24.11.2004

[www.AndreasKrueger.de/networks](http://www.AndreasKrueger.de/networks)  
networks@AndreasKrueger.de

V1.04

26.11.2004

networks@AndreasKrueger.de

1 of 43

## Übersicht über den “roten Faden”

### Derzeitiger Netzwerk-“Hype” der Physik:

- Graphen in Statistische-Mechanik Modellen
- Begriffsklärung, 3 wichtigste Graph-Maße
- “altes” Erdös-Renyi Zufallsgraphen Modell
- 1998: Paradigmenwechsel bei Zufallsgraphen
  - neue empirische Ergebnisse
  - neue Modelle für Zufallsgraphen
  - neue Community von Physikern

### ProjektNetzwerke für Analyse von EU-finanziertem R&D:

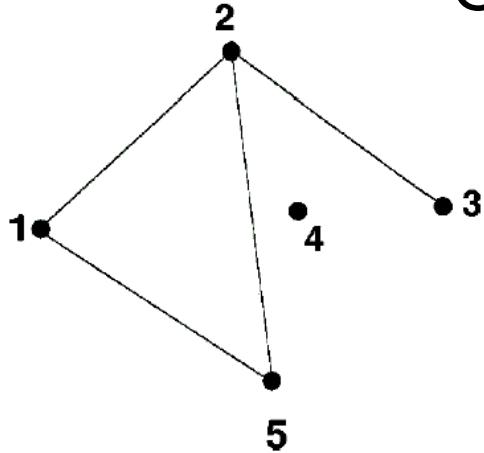
- Organisationen und Projekte bilden bipartite Graphen
- Projektionen auf unimodale Graphen
- Struktur unserer Zusammenarbeit aus 4 Fakultäten
- Modelle, Algorithmen
- *Allererste Simulations-Ergebnisse, Vergleich mit Empirie*

26.11.2004

networks@AndreasKrueger.de

2 of 43

# Graph $G=(V,E)$



- node, vertex, actor, point, individual ...
- edge, line, bond, tie, connection ...

$V \subseteq Z^+$  elements represented by dots

e.g.  $V=\{1,2,3,4,5\}$

$E \subseteq V \times V$ , elements represented by lines

e.g.  $E=\{(1,2), (1,5), (2,3), (2,5)\}$

Neighbours:

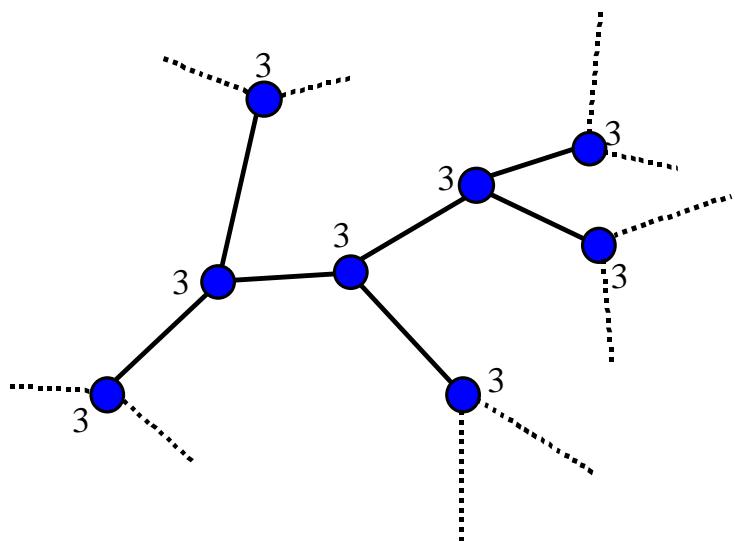
$x \sim y := \{\exists e \in E \text{ with } e=(x,y) \text{ or } e=(y,x)\}$

26.11.2004

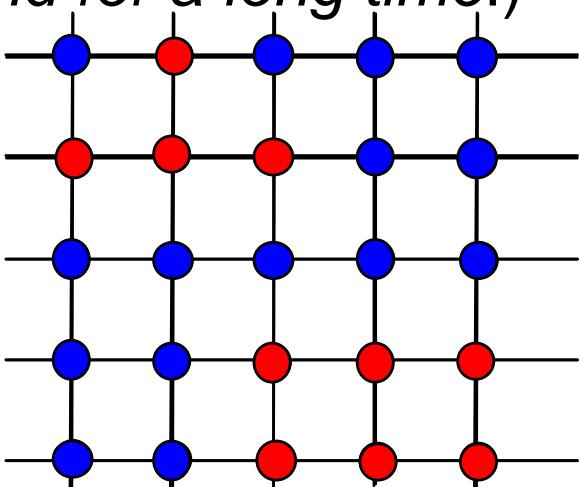
networks@AndreasKrueger.de

3 of 43

Special cases of graphs: Trees, Lattices  
are *regular graphs* (*around for a long time!*)



Caley-Tree with coordination number (degree)  $z=3$   
... branching process ...



Lattice  $Z^2$  - some processes:

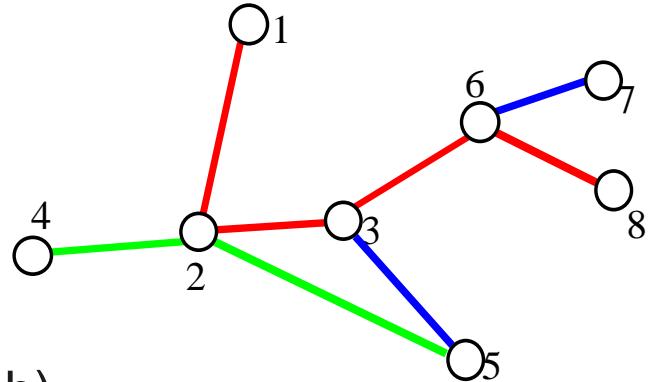
- condensed matter - crystals
- elementary particle theory
- self organized criticality (SOC)
- Ising (model magnet with spins)
- ...

26.11.2004

networks@AndreasKrueger.de

4 of 43

# graph measures 1: diameter, paths



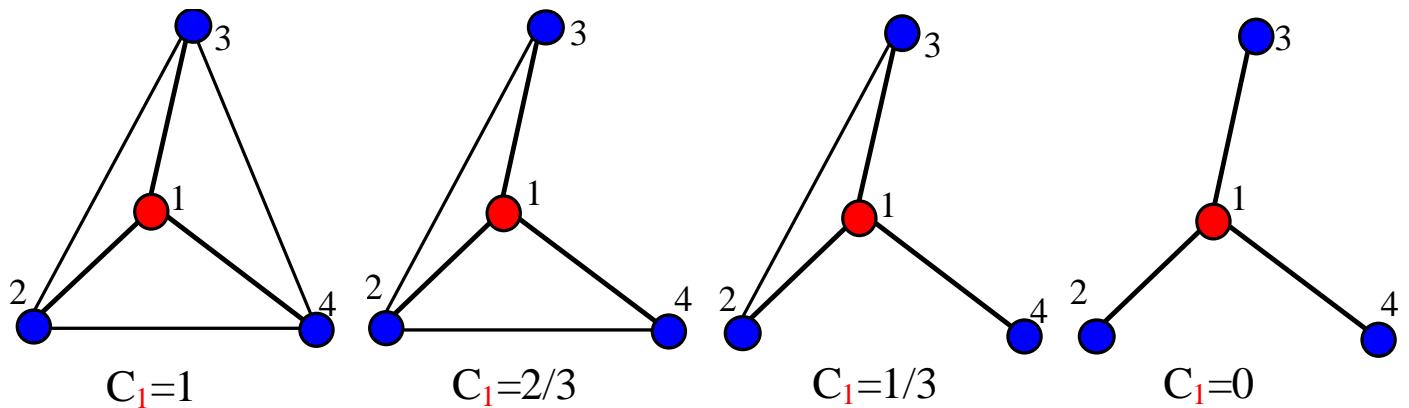
- pathlength (geodesic path)
  - **Shortest** connection between 2 nodes
  - Example **pathlength(1,8) = 4**
- global graph-properties
  - *Diameter* = **longest** geodesic path (here **4**)
  - *characteristic pathlength* = **average** of all paths  $(i,j)$

26.11.2004

networks@AndreasKrueger.de

5 of 43

# graph measures 2: Cluster-Coefficient, Triangle Number



$$C_i = \frac{\# T_i}{k_i(k_i - 1)/2}$$

$\# T_i$  = Number of Triangles around  
**vertex i**

$$C_{\text{global}} = \frac{1}{N} \sum_i C_i$$

$C_i$ : Estimator for **local density of connections**, “*how many of my friends are friends to each other?*”

26.11.2004

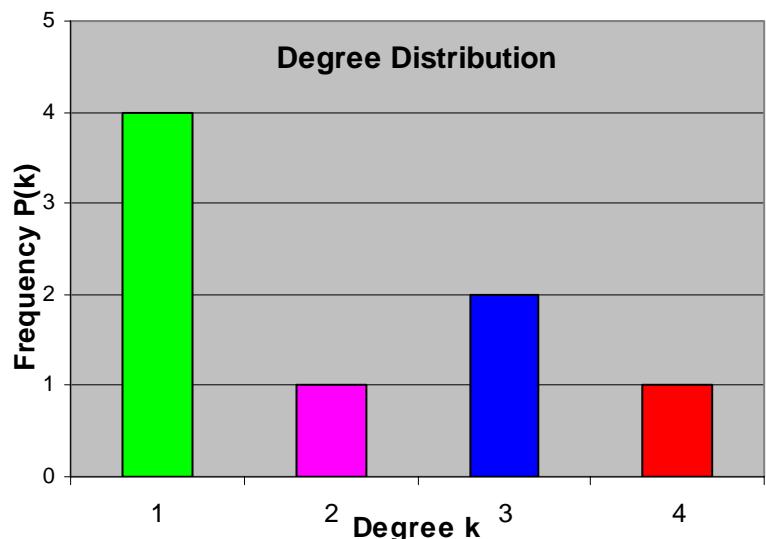
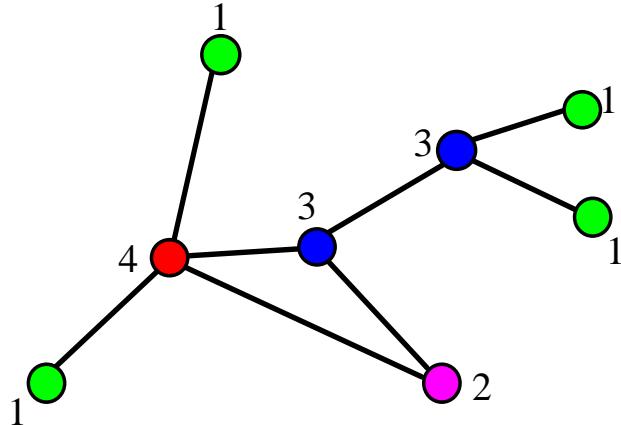
networks@AndreasKrueger.de

6 of 43

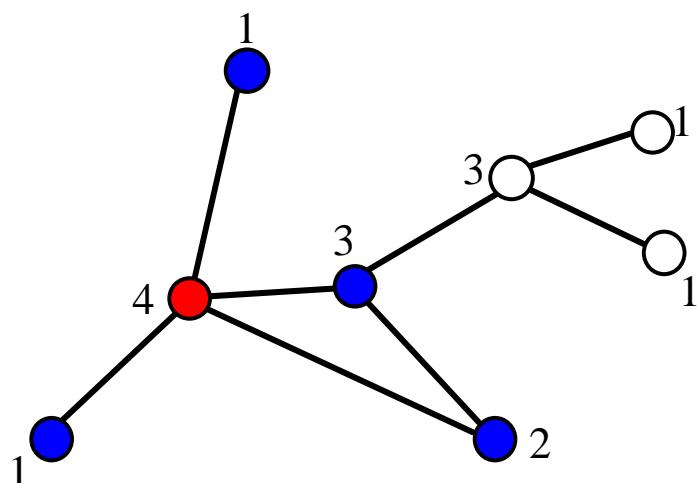
## graph measure 3: **degree** of a node

$k_x = \deg(x) = |N_1(x)|$   
= number of  $N_1$ -Neighbours of node x

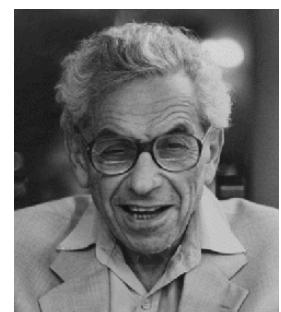
$P(k)$  = *Degree-Distribution (frequency)*  
= **number of nodes** with  $\deg=k$



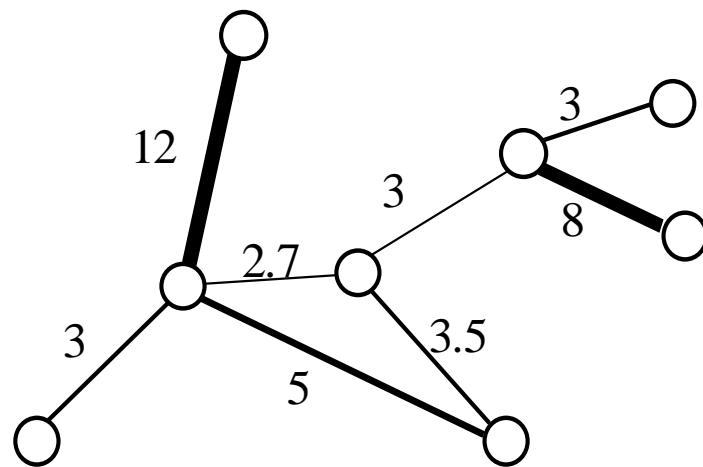
## Hubs and Authorities



- **Hub** ~ high degree
  - e.g. plane traffic: Chicago, Frankfurt Main
  - e.g. mathematics: Erdős
- **Authority** ~ linked by a Hub



# Gewichtete Kanten, gewichteter Graph



Funktion  $g(e)$  mit  $g: E \rightarrow |R$

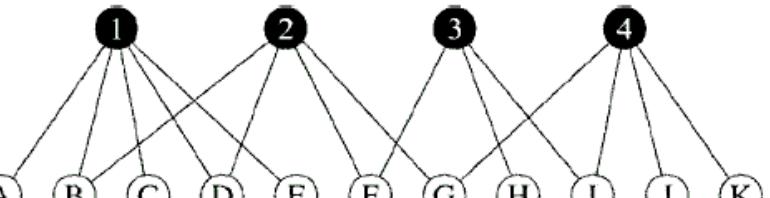
Gibt jeder Kante eine Stärke, Intensität, etc.  
(*ungewichtet*:  $g(e)=1$  für alle  $e$ )

26.11.2004

networks@AndreasKrueger.de

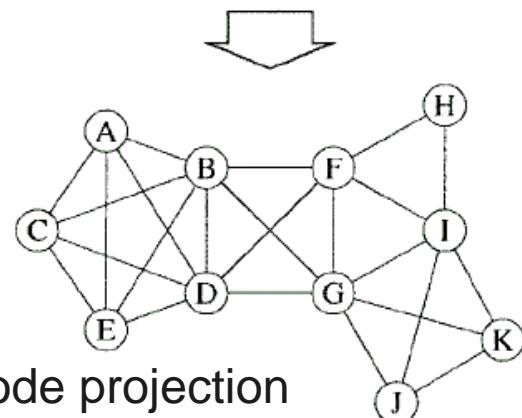
9 of 43

## Bipartite Graphs



Up to now we have only seen so called *1-mode graphs*, i.e. there is **one** type of vertices

Now imagine for example 4 **films** (black) and 11 playing **Actors** (white).



1-mode projection

From the 2-mode graph we can generate a 1-mode graph by projection (under information loss)

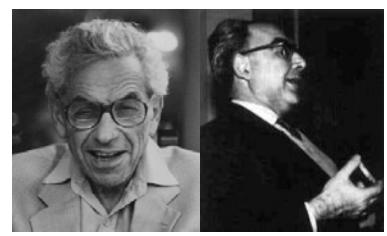
FIG. 14. A schematic representation of a bipartite graph, such as the graph of movies and the actors who have appeared in them. In this small graph we have four movies, labeled 1 to 4, and eleven actors, labeled A to K, with edges joining each movie to the actors in its cast. The bottom figure shows the one-mode projection of the graph for the eleven actors. After Newman, Strogatz, and Watts (2001).

26.11.2004

networks@AndreasKrueger.de

10 of 43

# THE random graph model: Erdös Renyi RandomGraph (~1960)

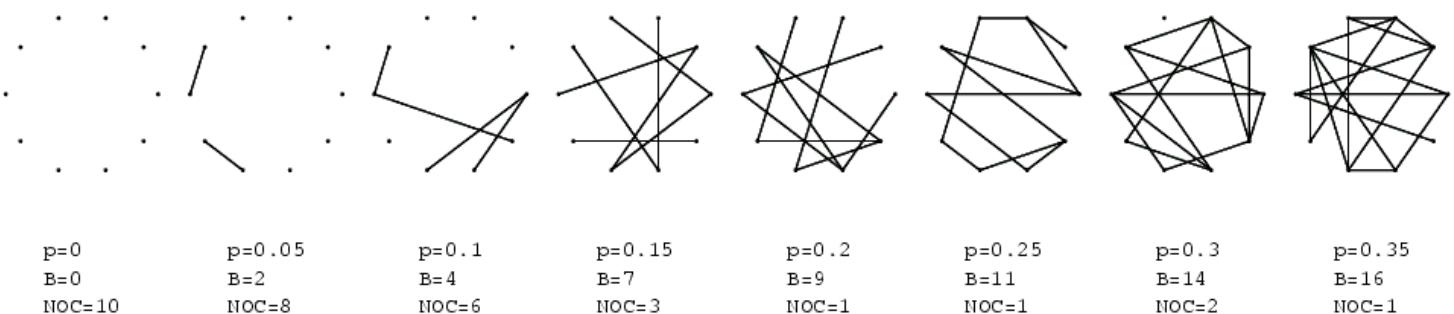


<http://www.nd.edu/~networks/linked/newfile6.htm>

- $G(N,p)$  random graph
- $N$  vertices  $\rightarrow$  # possible edges:
- $p$  independent probability for each edge (Bernoulli-process)

$$M_{\max} = \frac{N(N-1)}{2}$$

$$p \in [0,1]$$



26.11.2004

networks@AndreasKrueger.de

11 of 43

## Degree Distribution with ER $G(N,p)$ is $\sim$ Poisson

The average is good estimator  
for the whole distribution (bellshaped)

$$\langle k \rangle = (N-1)p$$

$$= (N-1) \frac{M}{N(N-1)/2} = \frac{2M}{N}$$

$$= \mu$$

The degree has a binomial distribution. For  $N \gg 1$  it becomes Poissonian:

$$P(k) = e^{-\mu} \frac{\mu^k}{k!}$$

with an exponential tail for large  $k$

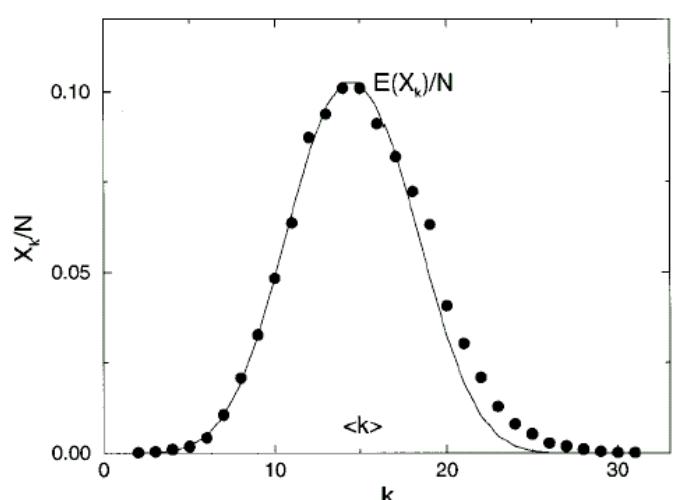


FIG. 7. The degree distribution that results from the numerical simulation of a random graph. We generated a single random graph with  $N=10\,000$  nodes and connection probability  $p=0.0015$ , and calculated the number of nodes with degree  $k$ ,  $X_k$ . The plot compares  $X_k/N$  with the expectation value of the Poisson distribution (13),  $E(X_k)/N=P(k_i=k)$ , and we can see that the deviation is small.

26.11.2004

networks@AndreasKrueger.de

12 of 43

# paradigm shift 1998/99

## static random → grown networks

### Starting Papers

- **Watts, D. J. and Strogatz S. H.,**  
*Collective dynamics of small-world networks,*  
**1998.06.04** Nature, 393, 440.
- **Barabasi, A.-L. and Albert, R.,**  
*Emergence of scaling in random networks,*  
**1999**, Science 286, 509–512 .
- Albert, R., Jeong, H. and Barabasi, A.-L.,  
*The diameter of the world-wide web,*  
 1999, Nature (London) 401, 130-131; cond-mat/9907038.
- Barabasi, A.-L., Albert, R., and Jeong, H.,  
*Mean-field theory for scale-free random networks,*  
 1999, Physica A 272, 173–187.
- Barabasi, A.-L.,  
**Linked: The New Science of Networks,**  
 Perseus, Cambridge, MA (2002).



Watts and Strogatz



Reka Albert



Albert-László Barabási



Hawoong Jeong

26.11.2004

networks@AndreasKrueger.de

13 of 43

### Property 1: small world

1998: Watts-Strogatz  
random rewiring

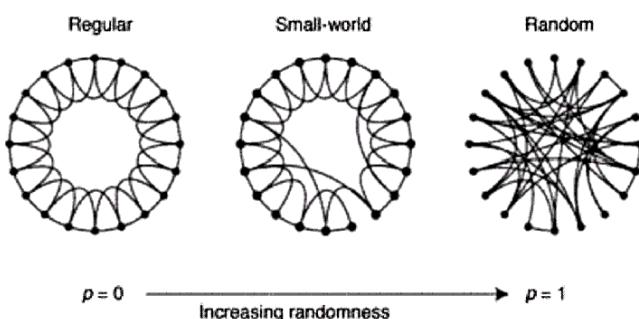


FIG. 15. The random rewiring procedure of the Watts-Strogatz model, which interpolates between a regular ring lattice and a random network without altering the number of nodes or edges. At  $p = 0$  all nodes, each connected to its four nearest neighbors, form a regular ring lattice. As  $p$  increases, the original ring is unchanged; as  $p$  increases further, the network becomes increasingly disordered until it becomes a random network at  $p = 1$ . After Watts and Strogatz,



1967: Milgram  
“6 degrees of separation”



<http://www.nd.edu/~networks/linked/newfile8.htm>

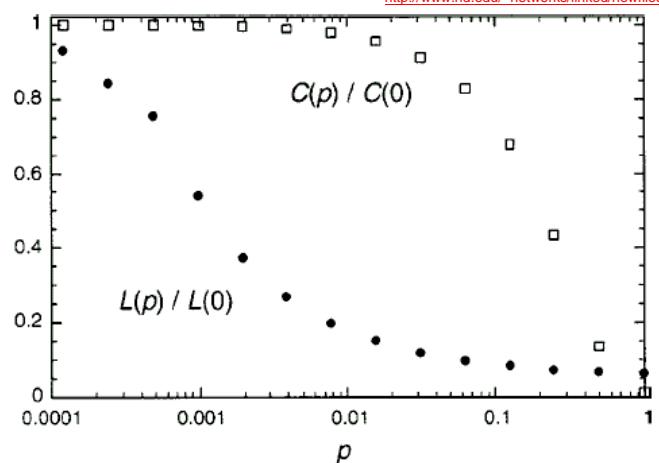


FIG. 16. Characteristic path length  $\ell(p)$  and clustering coefficient  $C(p)$  for the Watts-Strogatz model. The data are normalized by the values  $\ell(0)$  and  $C(0)$  for a regular lattice. A logarithmic horizontal scale resolves the rapid drop in  $\ell(p)$ , corresponding to the onset of the small-world phenomenon. During this drop  $C(p)$  remains almost constant, indicating that the transition to a small world is almost undetectable at the local level. After Watts and Strogatz, 1998.

$$\ell \sim \log N$$

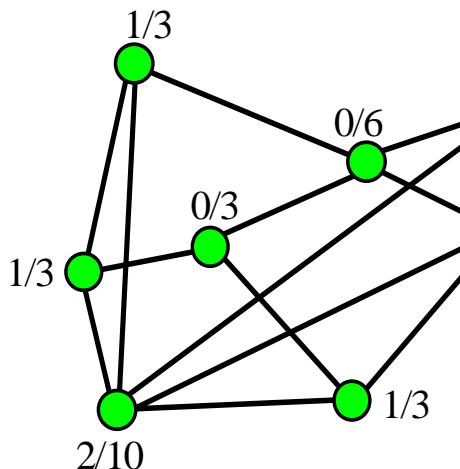
26.11.2004

networks@AndreasKrueger.de

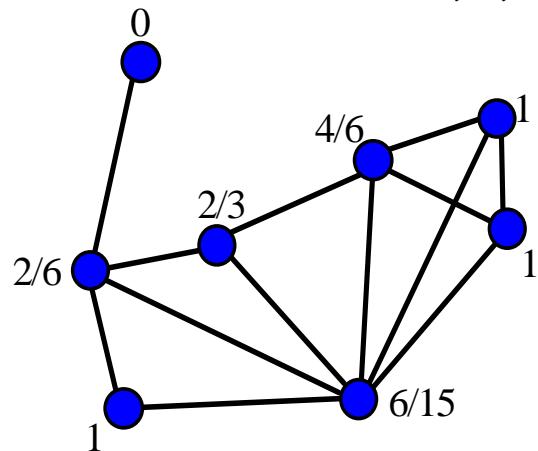
14 of 43

# Property 2: high clustering

$$C_i = \frac{\#T_i}{k_i(k_i-1)/2}$$



$$C=0.1917$$



$$C=0.6333$$

In both cases  $M=13$  and  $N=8$ , but in the *right* picture many more friends are themselves direct friends to each other ! “Empirical Networks” have a significantly **higher clustering-coefficient** than ErdosRenyi-RandomGraphs !

26.11.2004

networks@AndreasKrueger.de

15 of 43

# Property 3: scale free

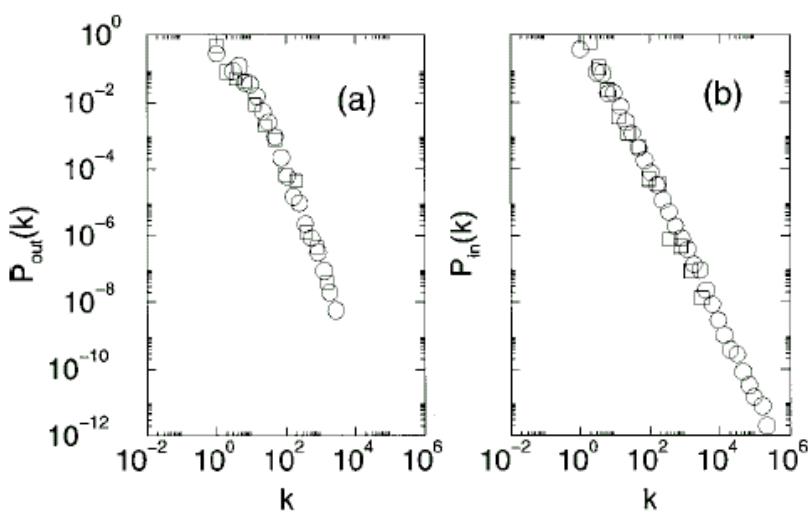


FIG. 2. Degree distribution of the World Wide Web from two different measurements:  $\square$ , the 325 729-node sample of Albert *et al.* (1999);  $\circ$ , the measurements of over 200 million pages by Broder *et al.* (2000); (a) degree distribution of the outgoing edges; (b) degree distribution of the incoming edges. The data have been binned logarithmically to reduce noise. Courtesy of Altavista and Andrew Tomkins. The authors wish to thank Luis Amaral for correcting a mistake in a previous version of this figure (see Mossa *et al.*, 2001).

In MEASURED networks, the degree distribution is not Poissonian (with exponential tail) for large  $k$  but "fat tail"  $\rightarrow$  falling power-law

$$P(k) = \frac{1}{k^\gamma}$$

$$\gamma \sim 2.5$$

An average  $\langle k \rangle$  doesn't really make sense here = no *built-in scale*

$\rightarrow$  „scale-free“

26.11.2004

networks@AndreasKrueger.de

16 of 43

# property 3: scale free

- Now an incredible run on real life data started...
- Almost identical scale-free distributions were measured in totally different objects, here e.g.
  - Internet Router
  - Actor-Movie-network
  - coauthors high energy physics
  - coauthors neuro sciences
- ! The measurements(!) almost perfectly lie on a straight line!**
- ! And the power-law exponents differ only a little!**

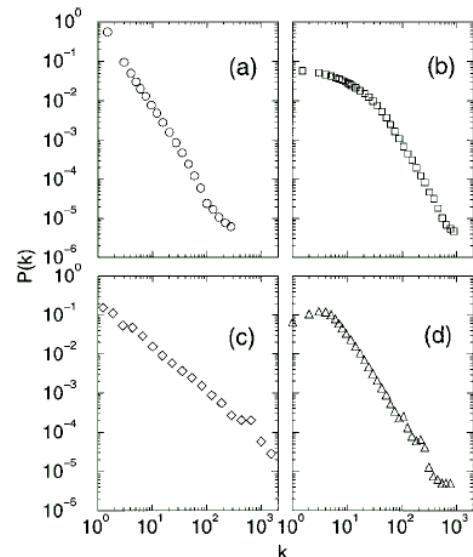


FIG. 3. The degree distribution of several real networks: (a) Internet at the router level. Data courtesy of Ramesh Govindan; (b) movie actor collaboration network. After Barabási and Albert 1999. Note that if TV series are included as well, which aggregate a large number of actors, an exponential cut-off emerges for large  $k$  (Amaral *et al.*, 2000); (c) co-authorship network of high-energy physicists. After Newman (2001a, 2001b); (d) co-authorship network of neuroscientists. After Barabási *et al.* (2001).

26.11.2004

networks@AndreasKrueger.de

17 of 43

## Property 3: scale free – Some objects that seem to have a scale-free degree

- WWW
- Internet-Routing
- Protein-Protein-docking
- citations
- collaborations
  - publications
  - Movie-Actor-Network
- Human Sexuality Networks
- Telephone calls
- brains
  - Caenorhabditis elegans*
  - Humans
- computer code
- The Word Web of language

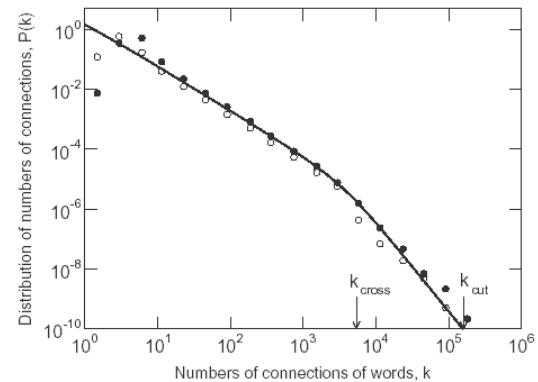


FIG. 9. The distribution of the numbers of connections (degrees) of words in the word web in a log-log scale [126]. Empty and filled circles show the distributions of the number of connections obtained in Ref. [126] for two different methods of the construction of the Word Web. The solid line is the result of theory of Ref. [127] (see Sec. IX) where the parameters of the Word Web, namely, the size  $t \approx 470\,000$  and the average number of connections of a node,  $\bar{k}(t) \approx 72$ , were used. The arrows indicate the theoretically obtained point of crossover,  $k_{cross}$  between the regions with different power laws, and the cutoff  $k_{cut}$  due to the size effect. For a better comparison, the theoretical curve is displaced upward to exclude two experimental points with the smallest  $k$  (note that the comparison is impossible in the region of the smallest  $k$  where the empirical distribution essentially depends on the definition of the Word Web).

26.11.2004

networks@AndreasKrueger.de

18 of 43

citations of the Phys. Rev. D <b>11-50</b> (1975-1994)	24,296	351,872	$\gamma_i = 3.0$	[27]
“——” (another fitting of the same data)			$\gamma_i = 2.6$	[99]
“——” (another estimate from the same data)			$\gamma_i = 2.3$	[94,95]
citations of the Phys. Rev. D (1982-June 1997)	—	—	$\gamma_i = 1.9$	[100]
collaboration network of movie actors	212,250	61,085,555	2.3	[55]
“——” (another fitting of the same data)			3.1	[102]
collaboration network of MEDLINE	1,388,989	$1.028 \times 10^7$	2.5	[13]
collaboration net collected from mathematical journals	70,975	$0.132 \times 10^6$	2.1	[15]
collaboration net collected from neuro-science journals	209,293	$1.214 \times 10^6$	2.4	[15]
networks of metabolic reactions	$\sim 500 - 800$	$\sim 1500 - 3000$	$\gamma_i = 2.2$	[41]
			$\gamma_o = 2.2$	
net of protein-protein interactions (yeast proteome) <sup>3</sup>	1870	2240	$\sim 2.5$	[44,45]
word web <sup>4</sup>	470,000	17,000,000	1.5	[126]
digital electronic circuits	$2 \times 10^4$	$4 \times 10^4$	3.0	[128]
telephone call graph <sup>5</sup>	$47 \times 10^6$	$8 \times 10^7$	$\gamma_i = 2.1$	[32]
web of human sexual contacts <sup>6</sup>	2810	—	3.4	[132]
food webs <sup>7</sup>	93 – 154	405 – 366	$\sim 1$	[48,49]

TABLE I. Sizes and values of the  $\gamma$  exponent of the networks or subgraphs reported as having power-law (in-, out-) degree distributions. For each network (or class of networks) data are presented in more or less historical order, so that the recent exciting progress is visible. Errors are not shown (see the caption of Fig. [24]). They depend on the size of a network and on the value of  $\gamma$ . We recommend our readers to look at the remark at the end of Sec. [VC2] before using these values. <sup>1</sup>The data for the network of operating AS was obtained for one of days in December 1999. <sup>2</sup>The value of the  $\gamma$  exponent was estimated from the degree distribution plot in Ref. [104]. <sup>3</sup>The network of protein-protein interaction is treated as undirected. <sup>4</sup>The value of the  $\gamma$  exponent for the word web is given for the range of degrees below the crossover point (see Fig. [9]). <sup>5</sup>The out-degree distribution of the telephone call graph cannot be fitted by a power-law dependence (notice the remark in Sec. [VF]). <sup>6</sup>In fact, the data was collected from a small set of vertices of the web of human sexual contacts. These vertices almost surely have no connections between them. <sup>7</sup>These food webs are truly small. In Refs. [50,51] degree distributions of such food webs were interpreted as exponential-like.

## First Model Albert Barabasi: **Preferential-Attachment**

### 1. Growth ! Not in static systems...

- Per time step **1 new node** and **m new edges**

### 2. Preferential Attachment

- **y~x:** new node y, an old node x, but which one?
- Probability to choose x linearly proportional to current degree of x:

$$P(\deg(x)=k_i) = k_i / \sum(k_i);$$

„the rich get richer“

- |     |                                |
|-----|--------------------------------|
| YES | → scale-free, exponent gamma=3 |
| YES | → small world property         |
| NO  | → high clustering              |

# This can only be a *very short glance*

- The whole subject was born out of the Internet, which is not a small but at least a *medium sized* system (e.g.  $\sim 10^{10}$  webpages) and can be measured much easier than nature or society
- Our (physics) explanatory approach usually goes for the „thermodynamical limit“ of  $N \rightarrow \infty$ ; very different from the sociological viewpoint
- Within 6 years, some 3000(?) papers have been published about networks in the physics community
  - pre-prints on [www.arxiv.org](http://www.arxiv.org) → cond-mat
- Many scientists leave their (neighbouring) fields and do research on networks now
- It resembles a little bit the hype of the 80ies „Fractals/Nonlinearity/Chaos,“ – everything was a fractal back then, now everything is networks
- Suggestions for further reading at the end of the talk ...

26.11.2004

networks@AndreasKrueger.de

21 of 43

## Teil 2: Projekt-Netzwerke

- Datensatz beim ARC systems research GmbH, Seibersdorf bei Wien
  - ca. 40,000 EU-geförderte Forschungsprojekte 1984-2004
  - Rahmenprogramm der EU dauern 4 Jahre; mittlerweile 2stellige Milliardenbeträge für Forschungs- und Entwicklungsförderung, Aber „nur“ ca. 6% aller Forschungsprojekte (Rest zB nationale Förderung)
  - Quelle: CORDIS-Datenbank der EU, Download, Standardisierung
- Was wissen wir über ein Projekt
  - Beteiligte Institutionen
  - Titel, Themen
  - Beginn, Dauer
  - Verantwortliche, Anschrift, Department/Faculty, ...
- **Grundidee:** DB → Netzwerk Es ist ein bipartiter Graph: Projekte  $\leftrightarrow$  Organisationen  
Dann unimodale Projektion: Projekte werden im Graph verbunden, wenn nicht-leere Schnittmenge von Organisationen
  - Projektion auf Projektgraph
  - Projektion auf Organisations-Graph
  - Frühes Ergebnis: **skalenfreie** Degree-Verteilung auf Organisationsgraph !!!

26.11.2004

networks@AndreasKrueger.de

22 of 43

# Set Graphs

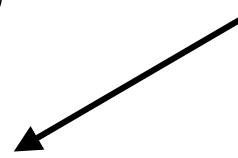
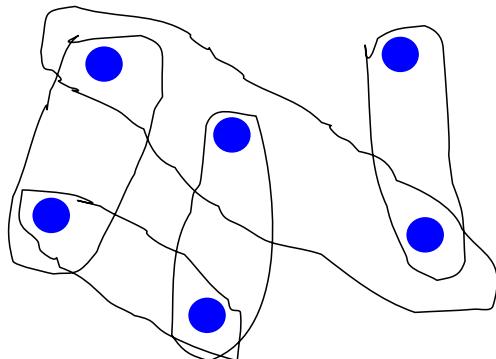
● = organization



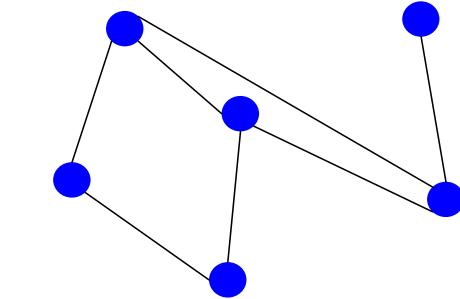
= project



● = project



bipartite  
graph



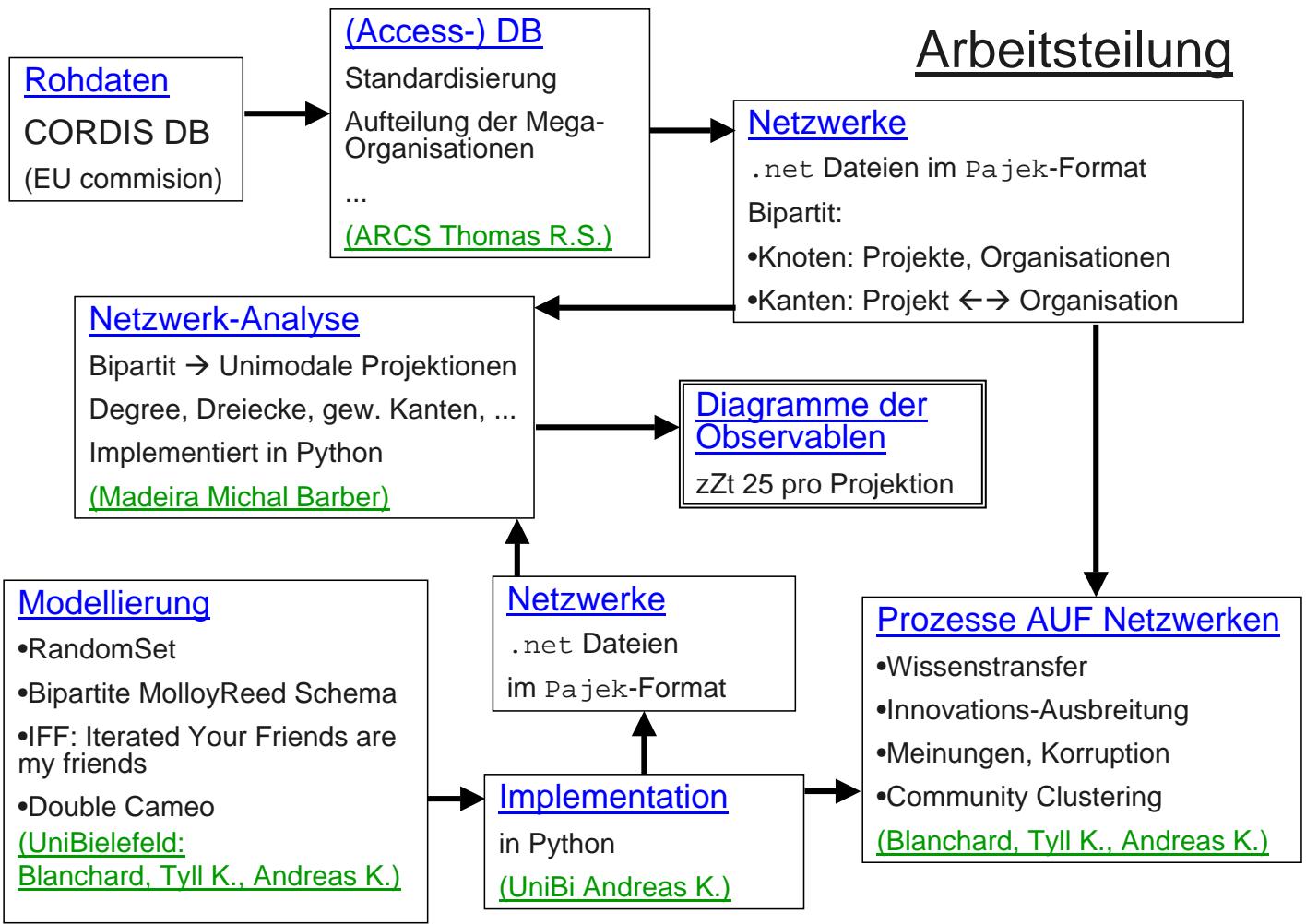
organization graph



project graph

## Projekt-Netzwerke

- unsere ersten Hypothesen zur Struktur-Entstehung:
  - Basismodell: *RandomSets* von rein zufällig zusammengewürfelten Mengen von Organisationen sind Projekte
  - Erweitertes IFF (Iteriertes your Friends are my Friends) – Prinzip: Projekt aus bisherigen Projekt-Bekanntschaften und  $N_2$ -Umgebung
- Nur einige Fragen, die uns leiten:
  - Rekonstruierbarkeit durch Simulation?
  - Wesentliche Eigenschaften des empirischen Netzwerkes
  - Nützliche neue Netzwerk-Observablen entwickeln
    - meso-Skala zwischen lokal und global
    - Dreieckszahl/Clusterkoeffizienten auf gewichteten Graphen
  - nach unterschiedlichen Rollen im Netzwerk untersuchen
    - Hubs, zentralste Knoten... (topologische Rollen)
    - stabile Akteurs-Konfigurationen
    - Knoten/Kanten mit Eigenschaften (Industrie vs. Wissenschaft, Agrar vs. Telekommunikation, Kontraktformen, etc.)
  - welche Partitionierung in Communities ergibt instruktive Einsichten?
  - Übersetzung der Netzwerk-Sichtweise in die Wirtschaftswissenschaften



26.11.2004

networks@AndreasKrueger.de

25 of 43

## Programmierung

- **Python:** freie Script-Sprache mit vielen fertigen Libraries und abstrakten leistungsfähigen Datentypen; 1991 in Holland gestartet, seitdem im Netz weiterentwickelt
- Da sowohl Michael Barber wie ich nun in Python programmieren, können wir zum Teil den fertigen Code gegenseitig verwenden!
- **Einige Teilaufgaben:** Histogramme generieren, laden, speichern, auf-addieren und entpacken; Verteilungen aufeinander tunen, Netzwerke aufbauen, ablegen, einlesen, abspeichern; Knoten entfernen, Re-Index-ing; Rankings sortieren; Graph-Traversals; Zufalls-Verteilungen, Random-Pairing von Knotenmengen, u.v.m.
- Schon ersten Prozess auf unseren Netzwerken implementiert: **Korruptions-Ausbreitung**  
Ph.Blanchard, T.Krüger, A.Krueger, P.Martin (FU Berlin)

26.11.2004

networks@AndreasKrueger.de

26 of 43

# Algorithmus 1: RandomSetModel

1. Zielgröße:  $P$  Projekte,  $O$  Organisationen
2. Wähle eine Projektgrösse  $|P_i|$  aus
  - zB random-Variable mit power law Verteilung, mit zB exponent –3, und cut-off für Maximalgröße  $\sim \text{Sqrt}(O)$
  - zB tatsächliche Projekt-Größenverteilung aus empirischer Untersuchung
3. Für das Projekt  $P_i$  wähle  $|P_i|$  zufällige Organisationen aus dem Organisations-Pool
4. Wiederhole 2. und 3. bis  $P$  Projekte kreiert wurden
5. Identifiziere eventuell unbenutzte Organisationen und lösche sie (d.h. Pool muss vorher etwas größer sein)
6. Speichere Netzwerk als bipartiten Graph zur Weiterverarbeitung

# Algorithmus 2: bipartite MolloyReed Schema

1. Zielgröße  $P$  Projekte,  $O$  Organisationen
2. Wähle alle Projekt-Größen  $|P_i|$  und alle Organisations-Größen (Projekte pro Organisation)  $|O_j|$  aus → Degree jedes Knotens
  - zB power-law Verteilungen mit Exponenten  $\lambda_O$  und  $\lambda_P$  und Minimal- und Maximalgrößen
  - zB gemessene Projekt- und Organisations-Größenverteilungen aus empirischer Untersuchung
3. Tune die beiden Verteilungen auf  $P$  und aufeinander, so daß die selbe Anzahl  $M$  von Kanten herauskommt
4. Erstelle für beide Knotentypen „virtuelle“ Knoten, d.h. vervielfältige einen Knoten so oft wie sein Degree angibt
5. Random Pairing: Bilde aus den „virtuellen“ Knoten  $M$  Zufalls-Paare (Organisation, Projekt)
6. Entferne im entstandenen Netzwerk eventuelle Doppelpaare
7. Speichere Netzwerk als bipartiten Graph zur Weiterverarbeitung

# The very first results:

We will always see  
*3 diagrams at a time*  
 to compare empirical  
 and generated graphs

## Empirical Network: „FP3“

3rd framework programme

O=9615 P=5529 M=31380

Project-Sizes: min=1 max=73 mean=5.6

Orgas-Sizes: min=1 max=138 mean=3.2

### RandomSet Network

- ~ same #orgs, #projs as FP3
- ~ same project sizes as FP3

### bipMolloyReed Network

- ~ same #orgs, #projs as FP3
- ~ same project sizes as FP3
- ~ same organization sizes as FP3

26.11.2004

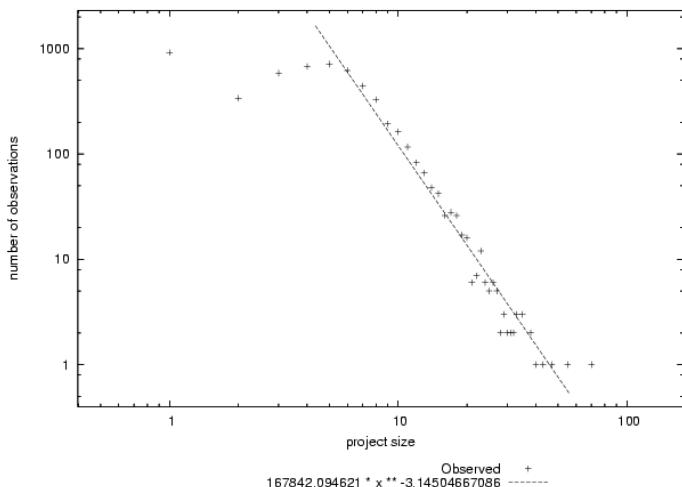
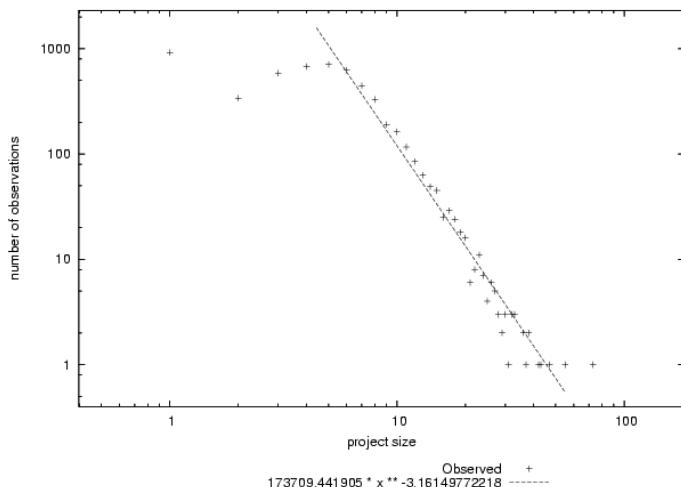
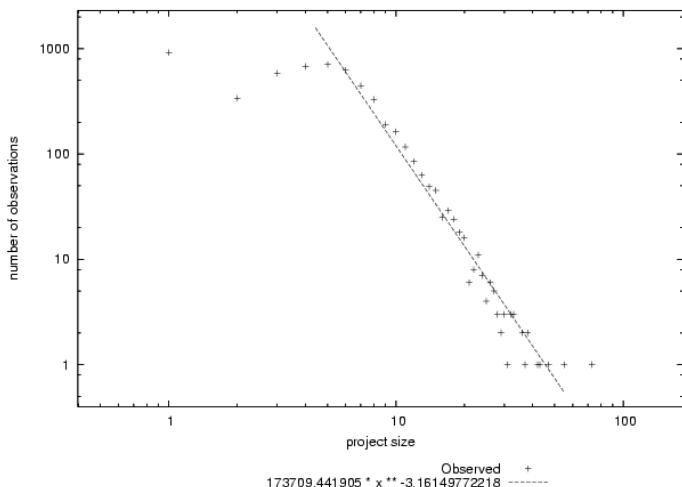
networks@AndreasKrueger.de

29 of 43

## Project Sizes:

identical

Because the empirical  
 project sizes are the inputs  
 for both simulations



26.11.2004

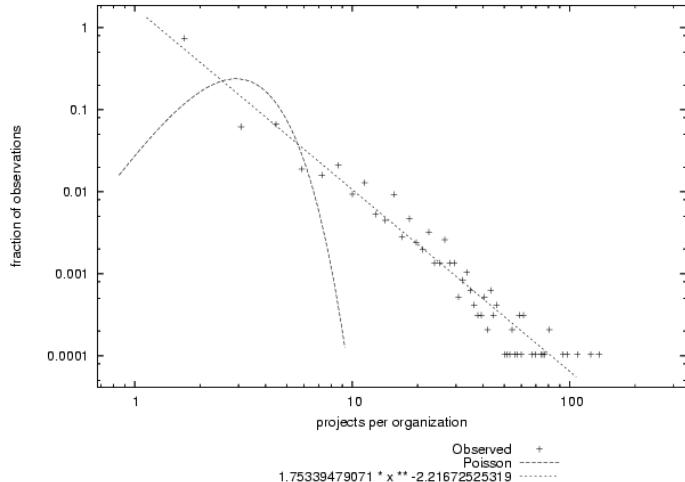
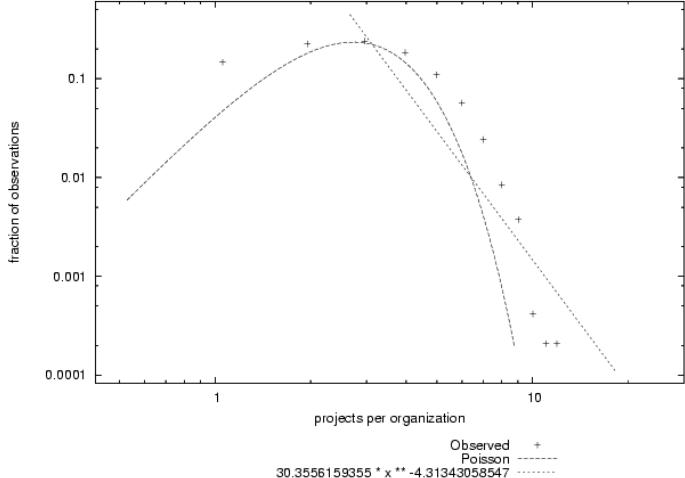
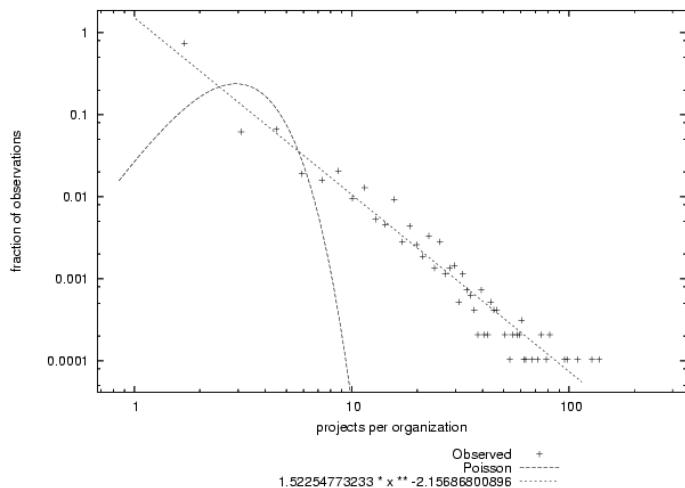
networks@AndreasKrueger.de

30 of 43

## Organization Sizes:

RandomSetModel:  
Exponential Decay ~ Poisson Distrib.  
predicted by theory!

FP3 <-> MolloyReed  
identical because input



26.11.2004

networks@AndreasKrueger.de

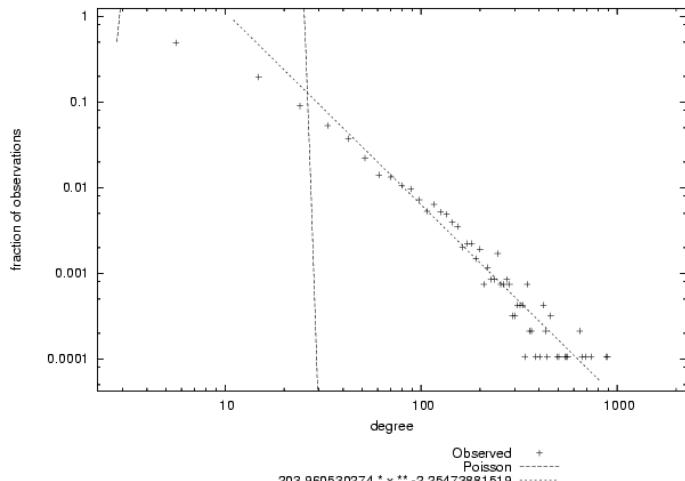
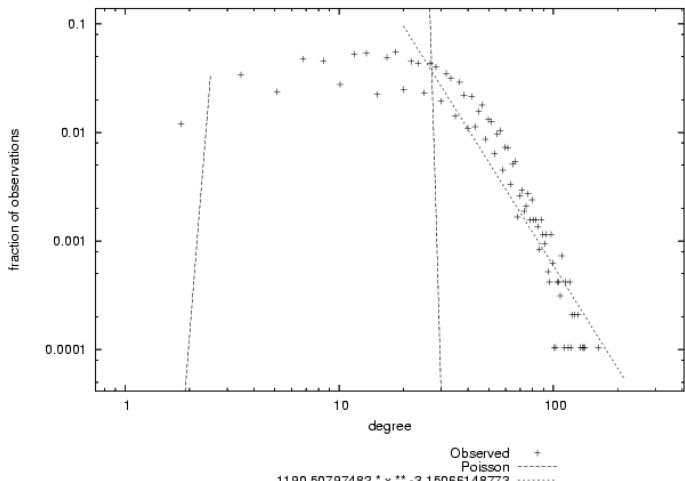
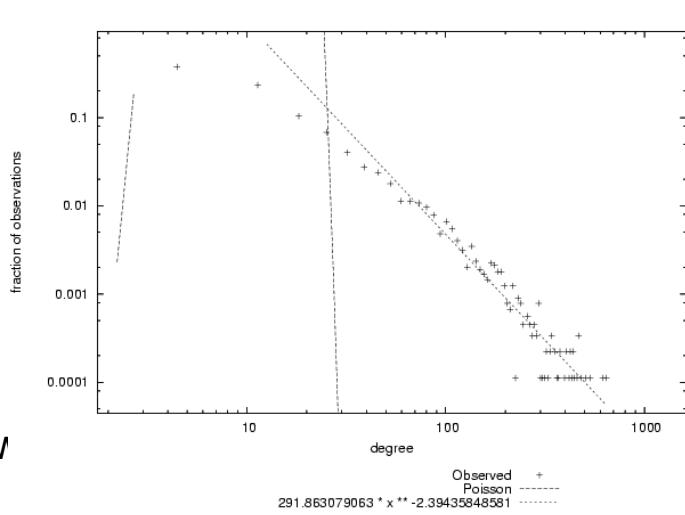
31 of 43

## Organization Graph:

Here the **degree distribution** partly correlates with the organization sizes

RandomSet:  
fat tail but much steeper

MolloyReed:  
similar to empirical FP3;  
both are almost on a straight power law  
with exponent 2.3 - 2.4



26.11.2004

networks@AndreasKrueger.de

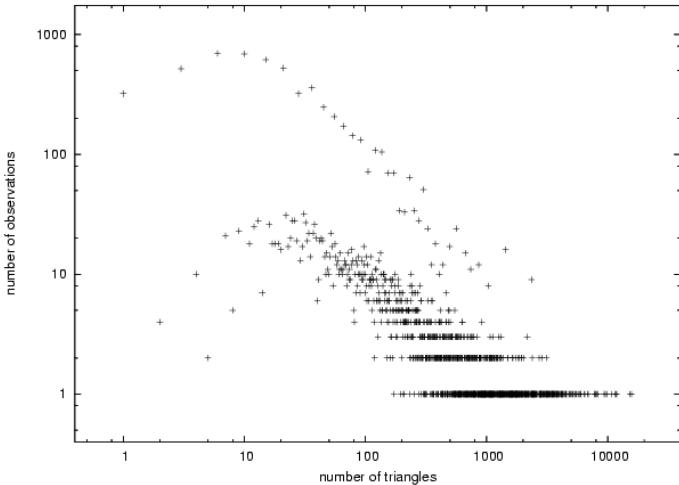
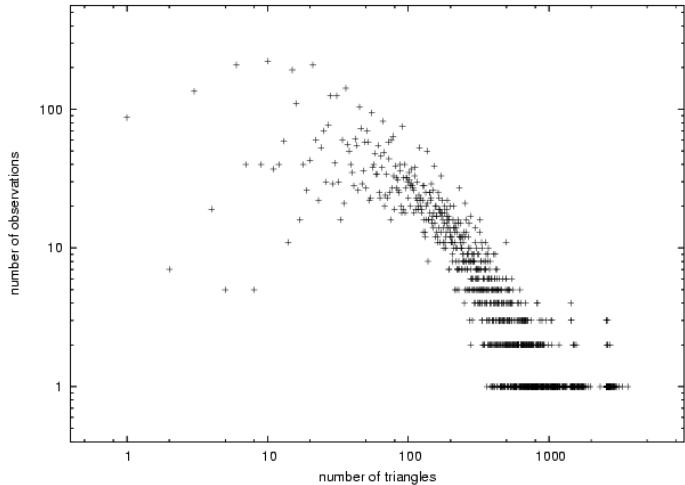
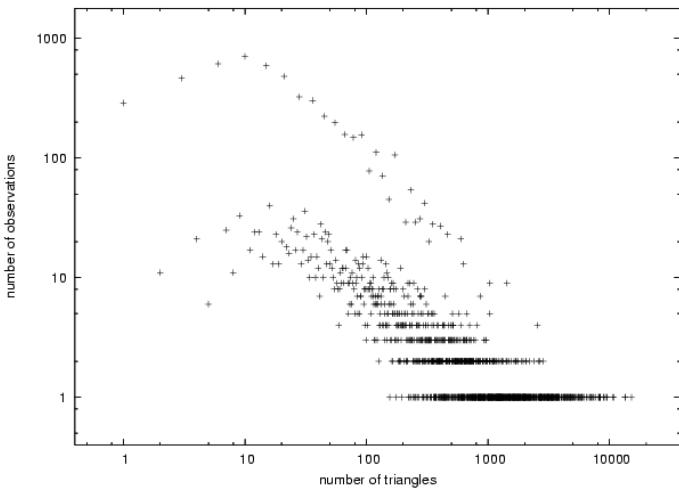
32 of 43

## Organization Graph:

The triangle count looks similar qualitatively in all three, but

### RandomSet:

lower highest-number-of-triangles  
lower highest-number-of-observations



26.11.2004

networks@AndreasKrueger.de

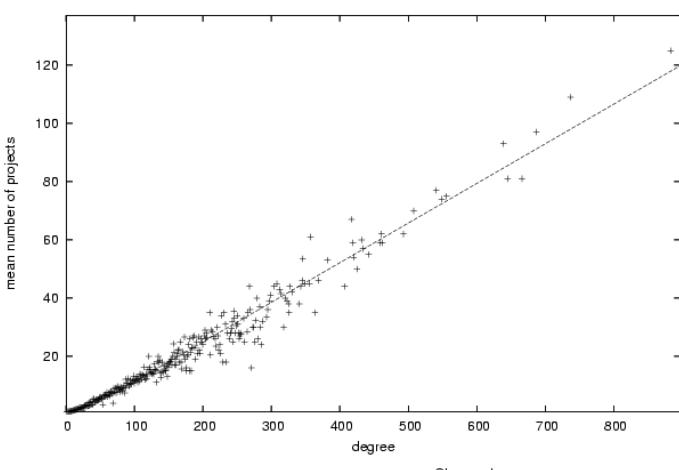
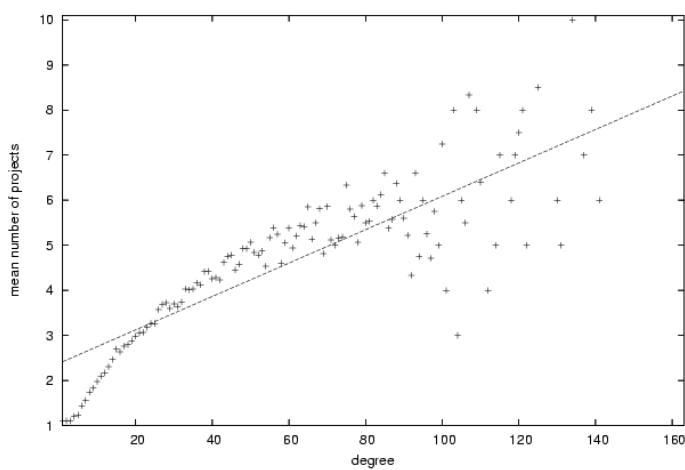
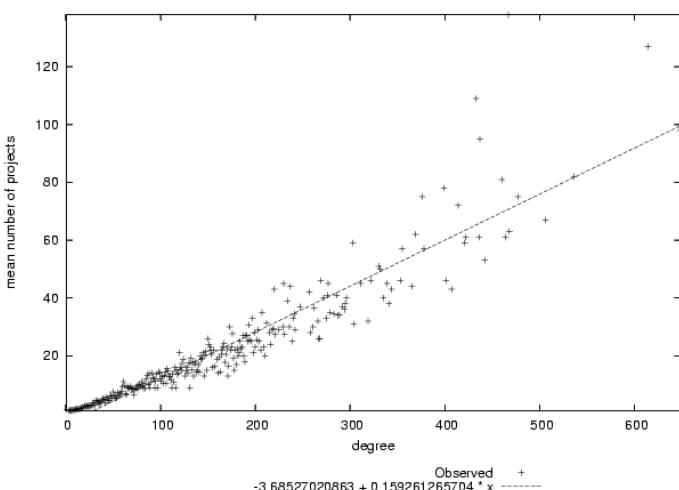
33 of 43

## Organization Graph:

multivariat:  
mean #projects vs. degree

### RandomSet:

totally different picture due to  
much smaller occurring degrees and  
mean number of projects per node



26.11.2004

networks@AndreasKrueger.de

34 of 43

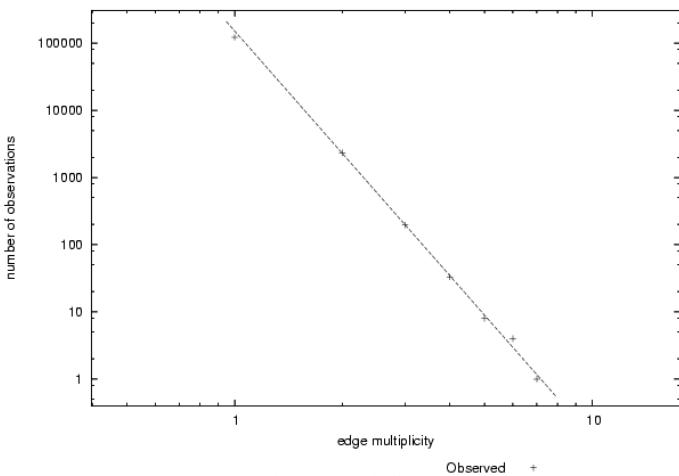
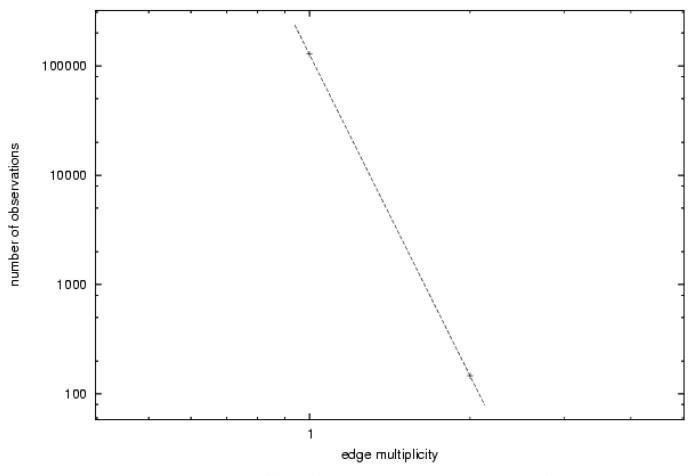
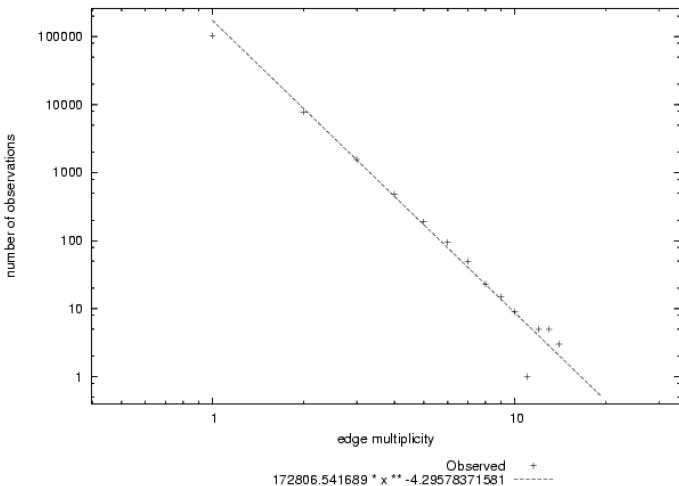
# Organization Graph:

## Edge Multiplicities

Empirical FP3:  
highest multiplicity 14

RandomSet:  
only 1 and 2

MolloyReed:  
highest multiplicity only 7



26.11.2004

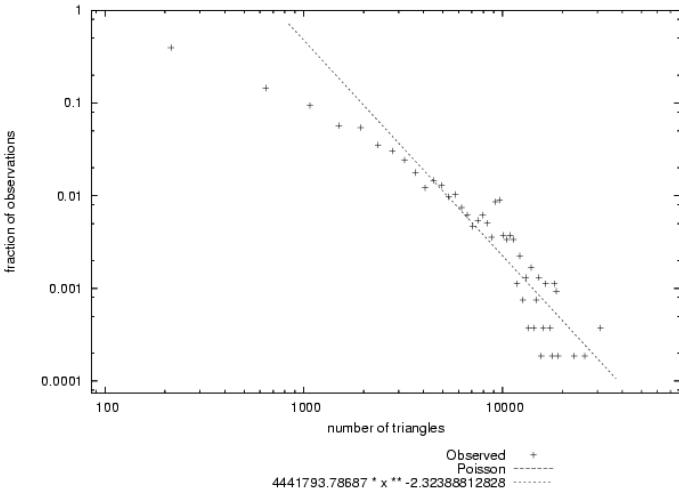
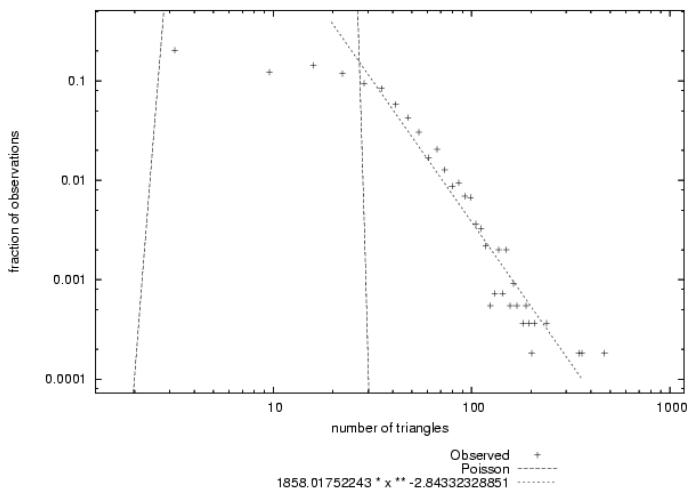
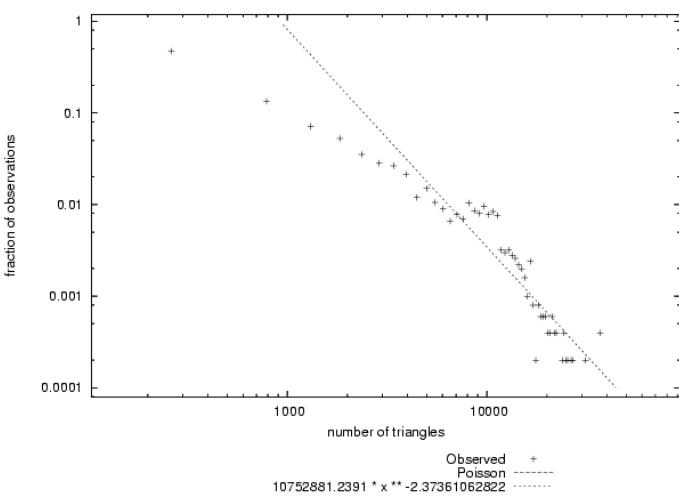
networks@AndreasKrueger.de

35 of 43

# Project Graph:

## Number of Triangles

RandomSet:  
lower highest-number-of-triangles,  
a little lower  
highest-number-of-observations  
→ much less triangles



26.11.2004

networks@AndreasKrueger.de

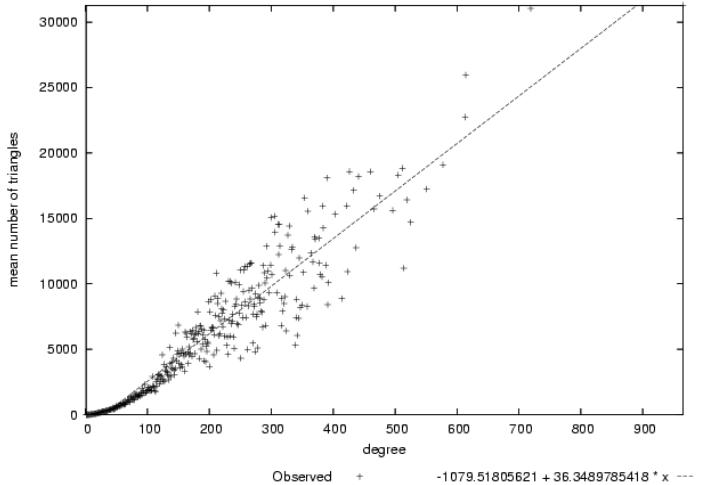
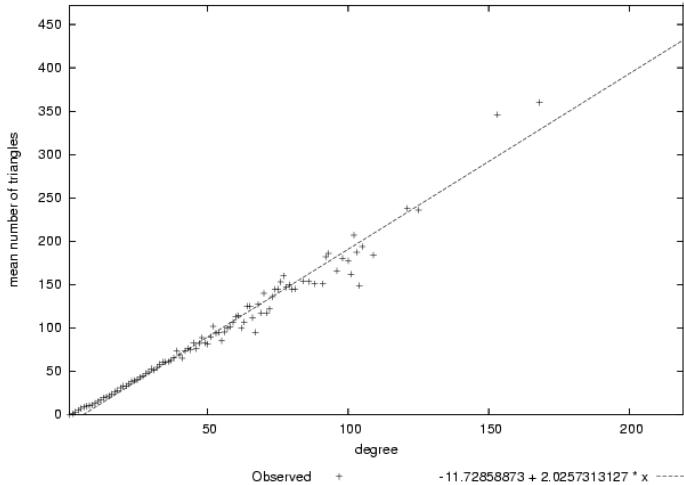
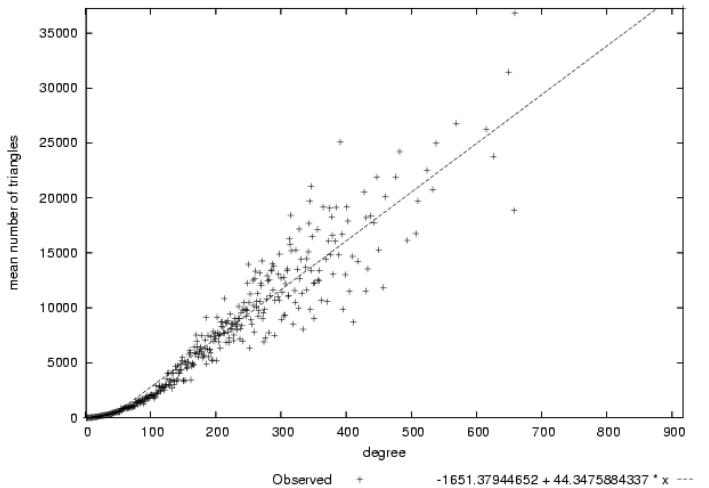
36 of 43

## Project Graph:

Mean triangle count

vs. degree

**RandomSet:**  
much lower degree  
and lower triangle counts



26.11.2004

networks@AndreasKrueger.de

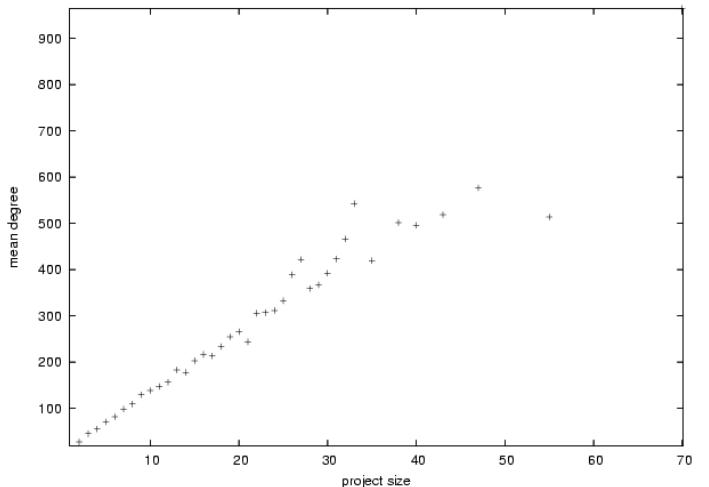
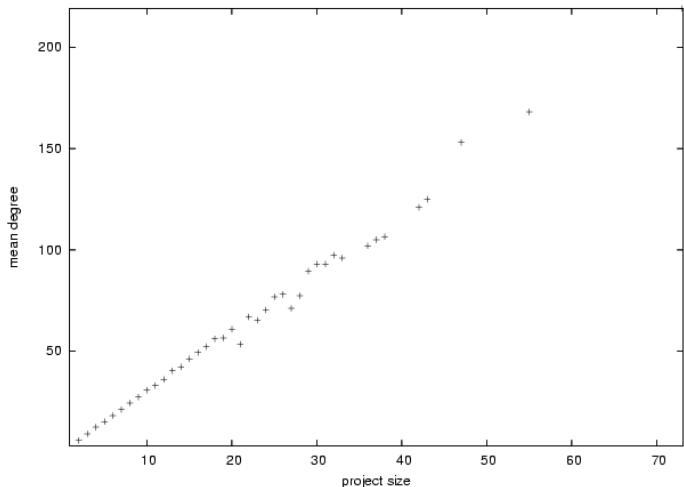
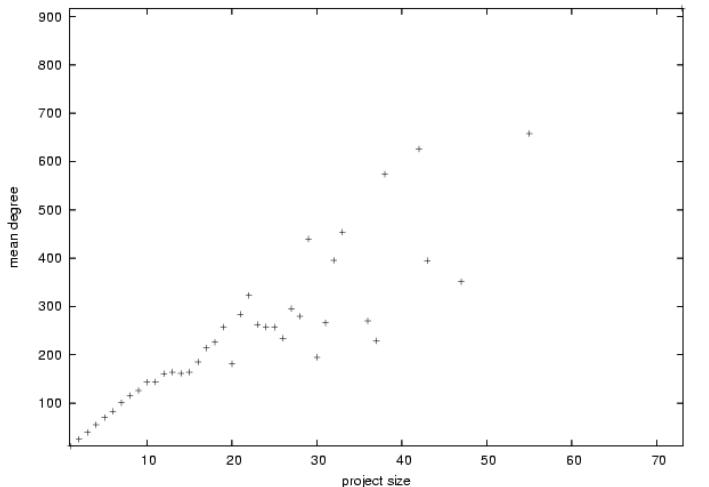
37 of 43

## Project Graph:

mean degree vs. project size

**RandomSet:**  
Much lower degree, so  
only the lower part of the diagram

**MolloyReed:** on straighter line than  
in more complex empirical result



26.11.2004

networks@AndreasKrueger.de

38 of 43

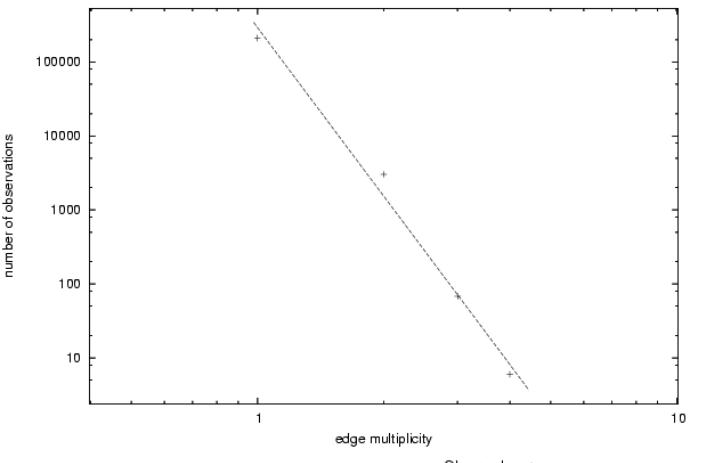
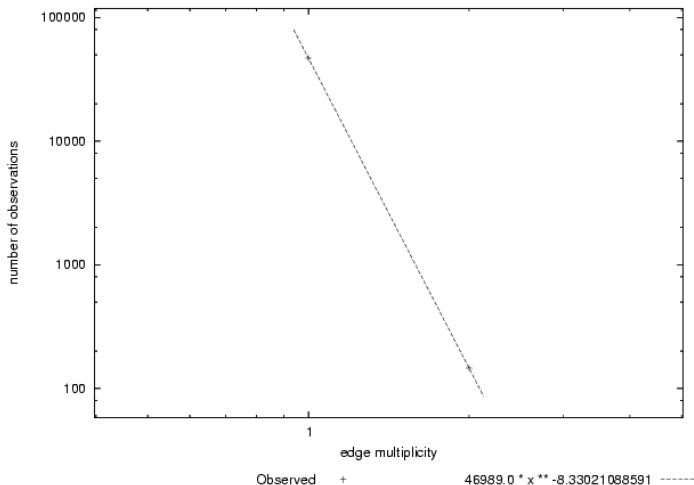
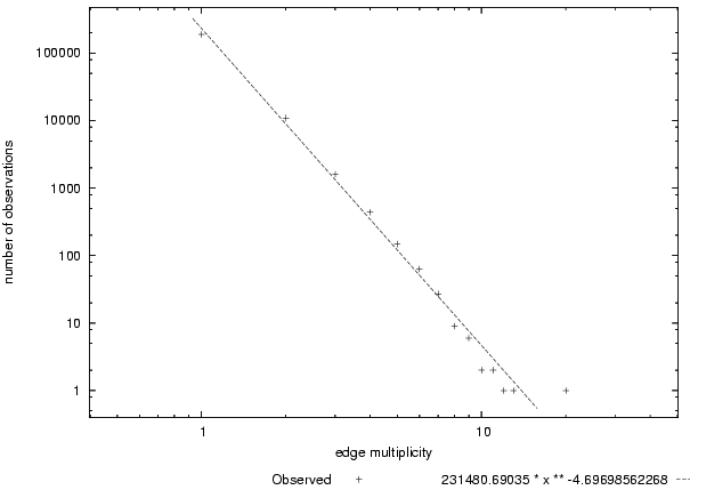
# Project Graph:

## Edge Multiplicities

Empirical FP3:  
highest multiplicity 20

RandomSet: only 1 and 2

MolloyReed: highest multiplicity 4



26.11.2004

networks@AndreasKrueger.de

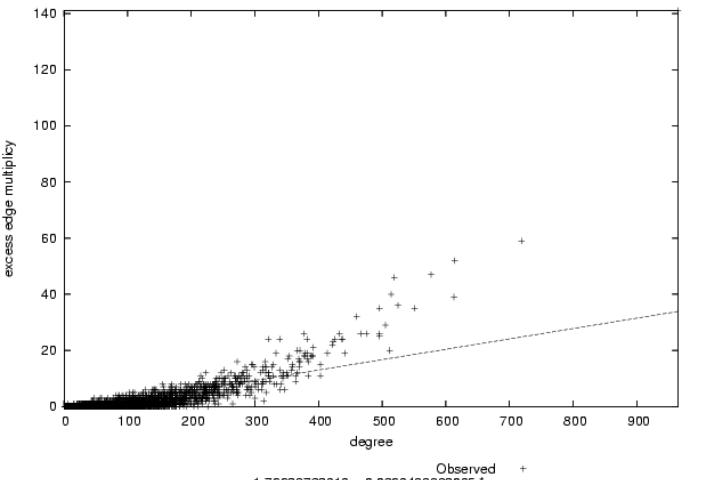
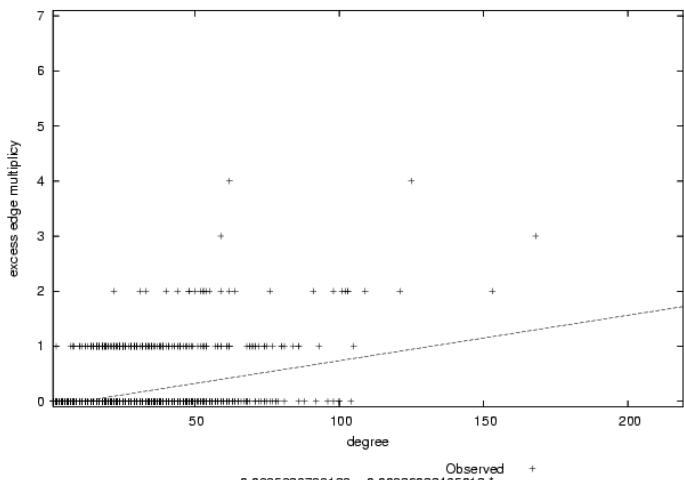
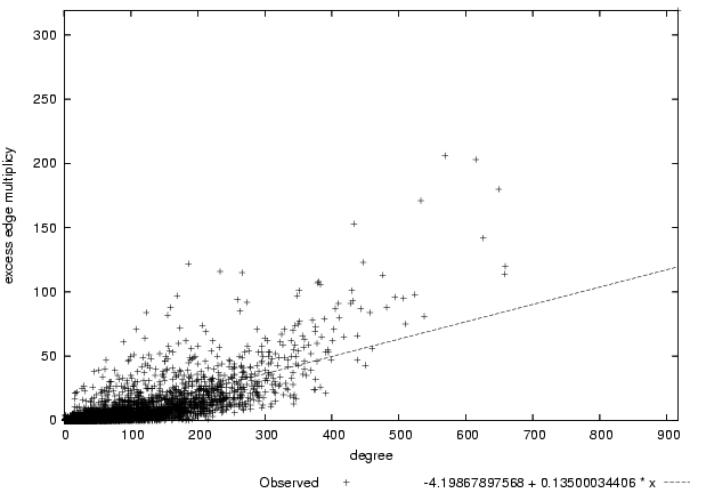
39 of 43

# Project Graph:

sum of excess edge multiplicities  
at vertices

RandomSet:  
only *very small* excess edge multiplicities

MolloyReed:  
Much smaller excess edge multiplicities  
than in empirical FP3,  
much more focussed on a line



26.11.2004

networks@AndreasKrueger.de

40 of 43

# nächste Modelle:

IFF = Iterated-Your-Friends-are-my-Friends Prinzip:

- Starte mit zusammenhängender Anfangsstruktur von Organisationen
- Würfle neues Projekt nur aus der  $N_1$ - und  $N_2$ -Nachbarschaft, d.h. früheren Projektpartnern und deren früheren Projektpartnern
- Iteriere solange, bis ?
- Erste Untersuchungen auf unimodalen Graphen haben gezeigt, daß eine skalenfreie Degree-Verteilung entsteht !  
(2003 Ph. Blanchard, T.Krueger, A.Ruschhaupt [cond-mat/0304563](http://cond-mat/0304563))
- und sowieso: lokale Vorschrift → hohes Clustering

Double Cameo:

Jede Organisation und jedes Projekt hat individuelle Parameter:

- Attraktivität  $\omega$ , Affinität  $\alpha$  für den Attraktivitätspараметer der anderen
- Attraktivität muss eine Seltenheitsverteilung  $\Phi(\omega)$  haben (zB falling power-law)
- Das Zusammenwürfeln geschieht umgekehrt proportional zur Häufigkeit  $\Phi(\omega_i)$  der einzelnen Attraktivität  $\omega_i$  hoch  $\alpha$ : 
$$P_{AB} \sim \frac{1}{[\Phi(\omega_B)]^{\alpha_A}}$$

Erste Untersuchungen zeigen, daß eine skalenfreie Degree-Verteilung entsteht und nur die „Extremisten“ in der Affinität den Exponenten bestimmen  
(2003/4 Ph. Blanchard, T.Krueger [cond-mat/0302611](http://cond-mat/0302611) und mit S.Fortunato [cond-mat/0407434](http://cond-mat/0407434))

# Projekt-Netzwerke

Zusammenfassend einige Ziele unserer Forschungskollaboration

- Publizieren ☺
- Theorien / Modelle / Methoden  $\leftrightarrow$  Empirie
- Interdisziplinäre Synergie-Effekte nutzen:  
Physik Mathematik Informatik Wirtschaft Systemwissenschaften
- neue Methoden nahe an der Anwendung entwickeln
- Unsere neue „Netzwerk-Physik“ anwenden und in die Wirtschaftswissenschaft / Systemanalyse transferieren:  
Was bedeuten diese mathematische Kenngrößen praktisch?
- Forschungsförderung in Europa:
  - Was ist eigentlich in diesen 20 Jahren passiert?
  - Fernziel: Maßnahmen-Katalog zur Auswahl von förderungswerten Projekten mit dem Blick auf das Gesamtsystem
  - Zum Beispiel:  
Förderung von Strukturen, die Innovations-Ausbreitung erleichtern!

THE END

# Vielen Dank für Ihre Aufmerksamkeit!

[networks@AndreasKrueger.de](mailto:networks@AndreasKrueger.de)  
[www.AndreasKrueger.de/networks](http://www.AndreasKrueger.de/networks)

## Lese-Empfehlungen:

### **Linked: The New Science of Networks,**

Barabasi, A.-L.; Perseus Cambridge MA (2002).  
Englisches Buch, sehr flockig geschrieben & leicht lesbar, "Prosa"

### ***Statistical Mechanics of Complex Networks***

Reka Albert and Albert-Laszlo Barabasi, 2001

[arXiv:cond-mat/0106096](https://arxiv.org/abs/cond-mat/0106096) ( [www.arxiv.org](https://www.arxiv.org) )

54 Seiten Review-Artikel, nicht mehr ganz neu, aber guter Einstieg