

GEP = Generalized Epidemic Process

Corruption as a GEP

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[arxiv:physics/0505031](https://arxiv.org/abs/physics/0505031)

My doctorate thesis consists of:

- Introduction into Complex Networks & SNA (incl. "ethics" chapter)
 - 1. The Network of EC-funded R&D projects (CORDIS database)
 - 2. "Corruption" – simulation of a contagion process on networks
 - 3. CAMBO clustering – finding clusters in networks by matrix reordering
- www.AndreasKrueger.de/networks → dissertation/disputation

Corruption?

Imagine any contagion process with

1. Neighbour infection
 - **Threshold** contagion, i.e. local infection **only if** „level of corruption of my neighbours exceeds Δ “
 - plus small infection probability if less than Δ
2. Mean field infection
3. Mean field disinfection

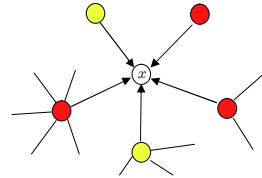
e.g.:

- opinions, fashions, ...
- waves of scientific hypes, discussed topics...
- processes of innovation diffusion, **knowledge diffusion**
- **Corruption...**

Corruption state variables

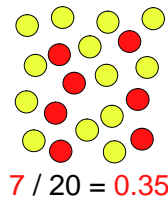
$\omega(x, t)$ in $[0, 1]$
 = node x is corrupt/non-corrupt at time t

$\Omega(x, t)$
 = number of infected neighbours
 = $\sum_{y \sim x} \omega(y, t)$



b_t = total prevalence of corruption at time t

$$b_t = \frac{1}{|V|} \sum_{y \in V} \omega(y, t)$$



Implemented LOCAL Processes:

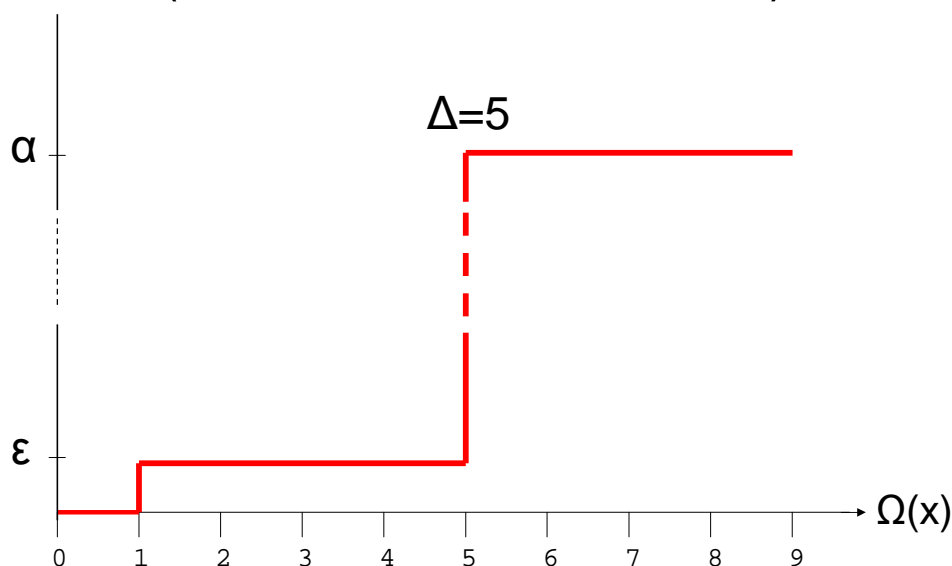
α -process: “if enough neighbours...”

- The **local** transmission probability for # of corrupt neighbours $\geq \Delta$
- Typical value: $\alpha \gg \epsilon, \beta, \gamma$
- Possible translations:
Influenceability by others, Decisiveness

ϵ -process: “if at least one neighbour ...”

- (similar to classical) **local** epidemic probability for # of corrupt neighbours $< \Delta$
- Typical value: $\epsilon \ll \alpha, \beta, \gamma$ (very small)
- Possible translations: **Naivety, Openness**

Local infection probabilities, (absolute $\Delta=5$ threshold)



Implemented GLOBAL Processes:

β -process: “infection through public opinion”

- The mean field transmission process due to the total (believed) prevalence of corruption
- Typical value: $\epsilon < \beta < \gamma$
- Possible translations:

**“Random” infection: How informed are you?
How much do you belief in mass media?**

$$\Pr_{\beta}(\omega_{t+1} = 1 \mid \omega_t = 0) = \beta(b_t)(1 - (1 - b_t)) = \beta(b_t)^2$$

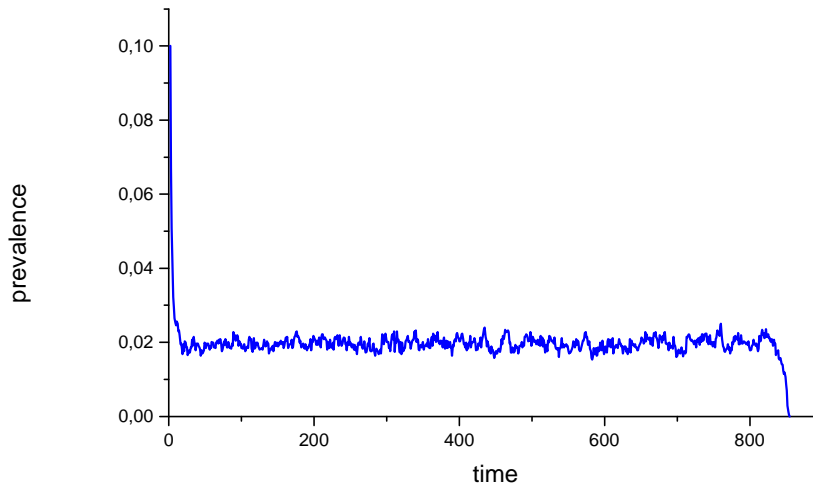
γ -process: “(only) the healthy can cure others”

- The (mean field!) corruption recover process due to the fight of the (healthy) society against corruption
- Typical value: $\beta < \gamma < \alpha$
- Possible translations:
random resistance / recovering / cleaning

$$\Pr_{\gamma}(\omega_{t+1} = 0 \mid \omega_t = 1) = \gamma(1 - b_t)$$

Single run until stagnation 1

Low semistable prevalence in a real collaboration network

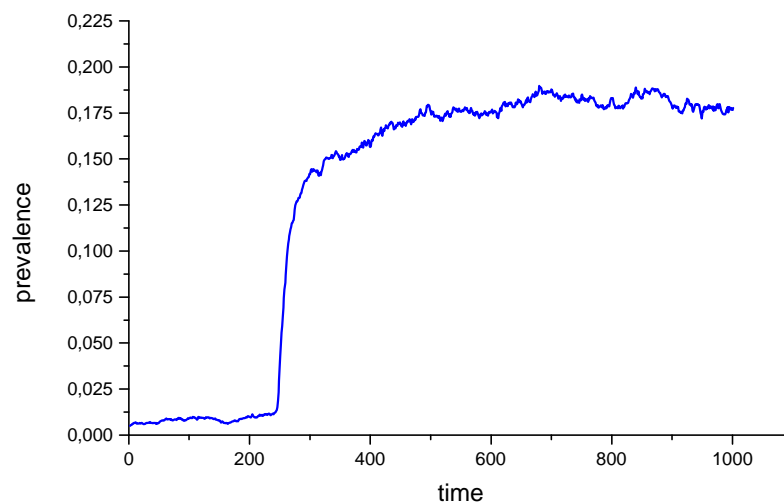


$$\Delta=30 \quad \varepsilon=0 \quad \alpha=0.99 \quad \beta=0.09 \quad \gamma=0.545 \quad b_0=0.10$$

Network FP2: N=4879 M=57633 mean degree=23.6 mean triangles=256.9

Single run until stagnation 2

Jump from very low prevalence to low prevalence

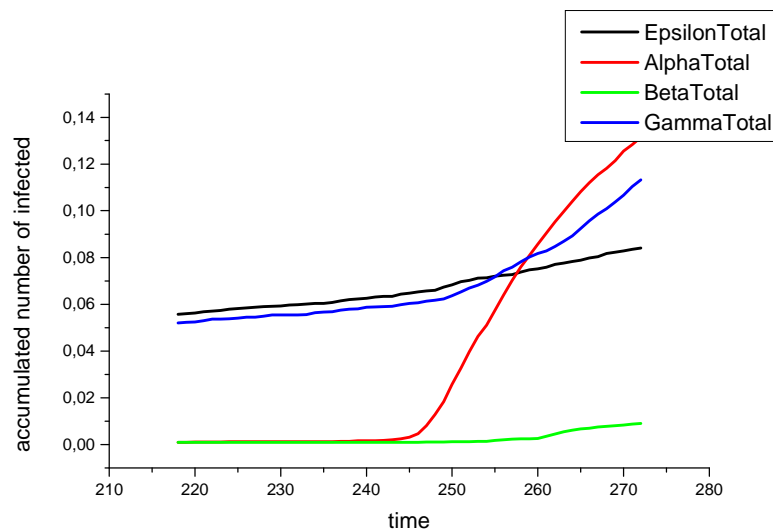


$$\Delta=25 \quad \varepsilon=0.001 \quad \alpha=0.20 \quad \beta=0.04 \quad \gamma=0.03 \quad b_0=0.005$$

Network FP3: N=7710 M=93852 mean degree=24.3 mean triangles=418.1

Single run until stagnation 2

Contribution of the 4 (des)infection paths for the “jump”



...(end of) single runs: some insights

- The Single-Process plots are very useful to understand the rich behaviour
- Exists possibility for a fatal resonance between local (ϵ , α) and mean-field process (β)
- small accumulation of infection unnoticed until a critical density (a point of no return) of corruption is reached
→ then quickly complete saturation of society

Similar to other complex systems with hidden phase transitions (e.g. climate)

Structure of the Python Program

Initial infection with b_0 corrupt nodes (random/ball)

→ update one vertex

→ all vertices, sync update

→ do until stagnation

Example Legend

get: end prevalence (usually ~ 0 or ~ 1)

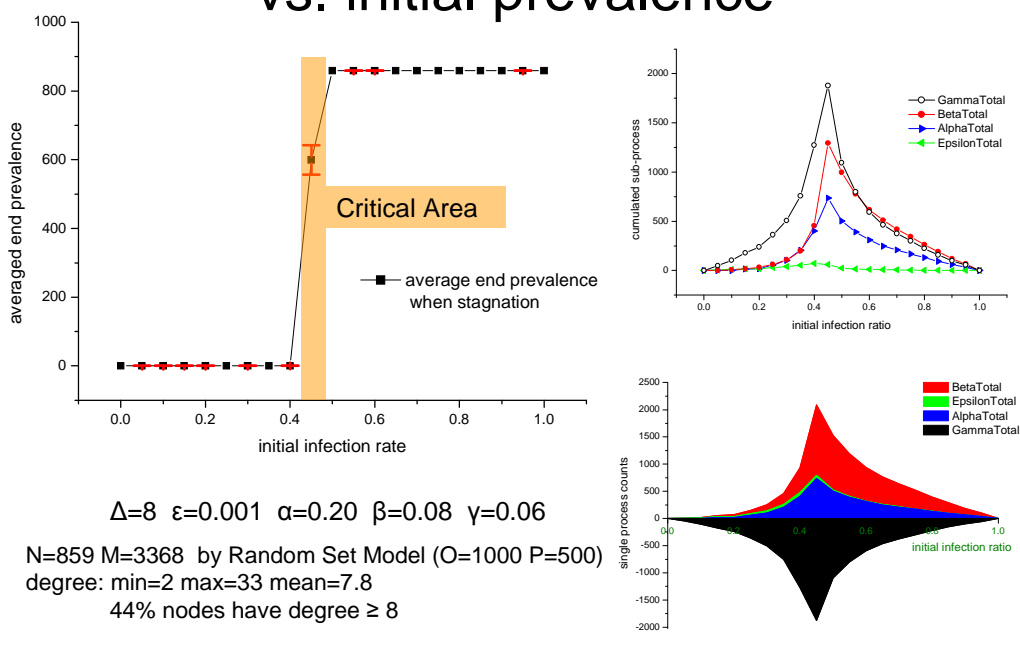
→ many runs to get *average* end prevalence

→ Transition finder: vary b_0 to locate $b_{\text{undercrit}}$ and b_{overcrit} and get (mean value) b_{crit}

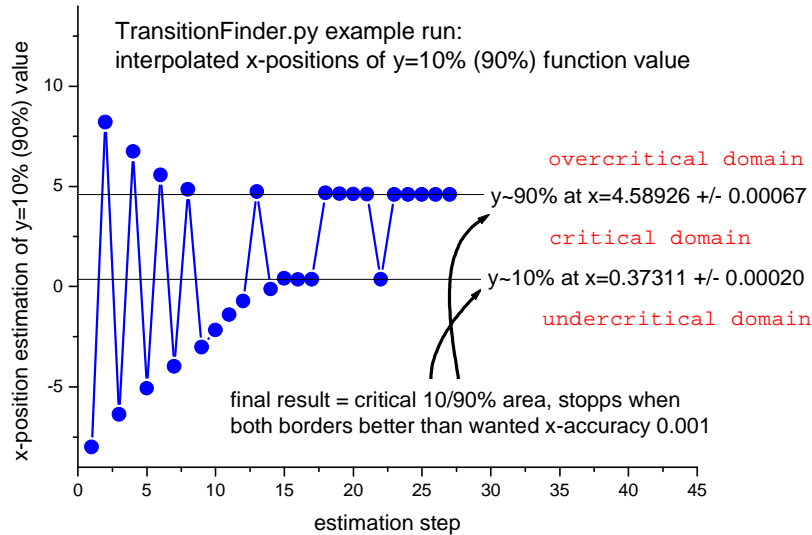
→ sweep (network property)

$X=N, M, T$ or λ to plot b_{crit} over X

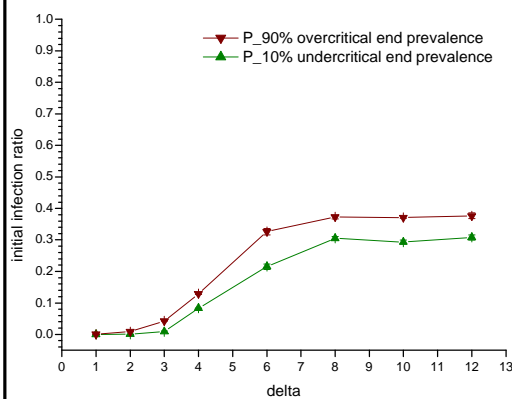
end prevalence vs. initial prevalence



Find x_{crit} but avoid critical area by linear interpolation of 10% / 90% y-value



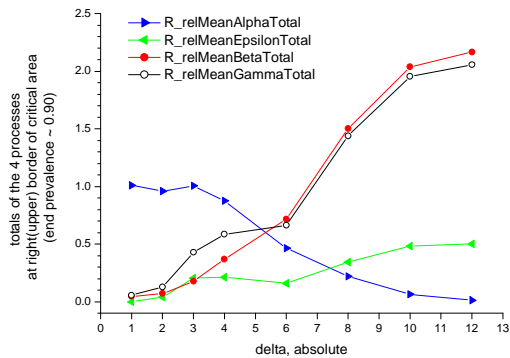
Critical Density over $\Delta =$ neighbour infection threshold



Lower and upper bounds for b_{crit} as a function of neighbour infection threshold Δ

$\epsilon=0.005$ $\alpha=0.35$ $\beta=0.08$ $\gamma=0.04$
GNM with $N=1500$ $M=5000$
→ mean degree ~ 6.7

Total number of state changes splitted into the 4 different sub-processes



Phase transition with respect to Initial Number of Infected Nodes

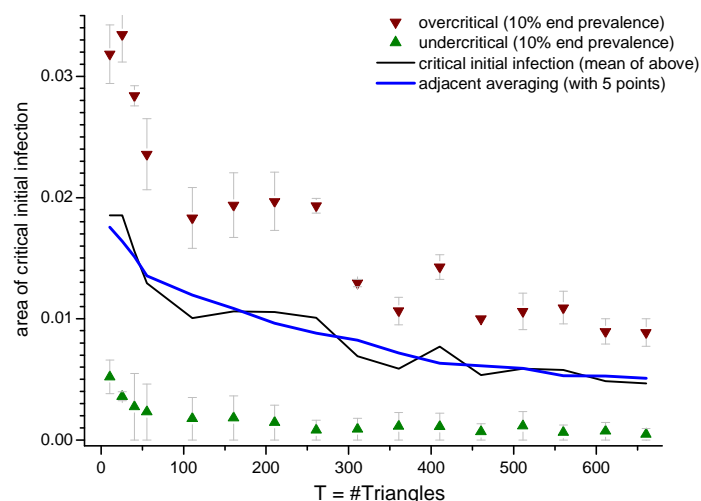
In corruption infection:

- both (mean-field and local) processes can have phase transitions **with respect to the *initial density of corrupt vertices***

Classical epidemic process is:

- either overcritical (reproduction number $R_0 > 1$)
→ and a *single initially infected node* infects a positive fraction with positive probability
- or undercritical ($R_0 < 1$)
→ all infected will die out, everyone's healthy

Critical Density over $T = \#$ triangles

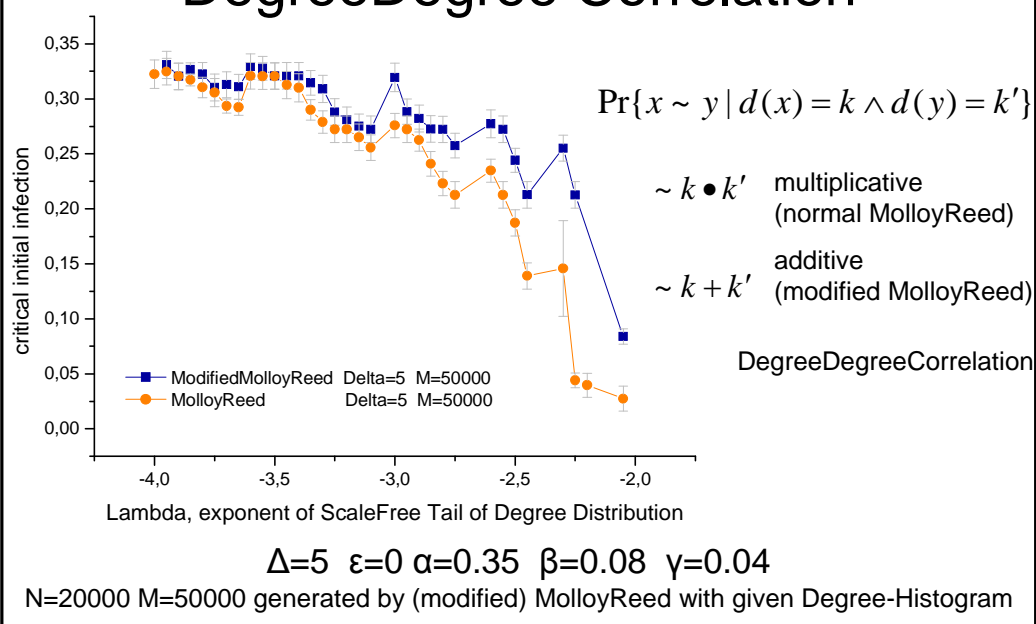


$\Delta=2$ $\epsilon=0.005$ $\alpha=0.30$ $\beta=0.08$ $\gamma=0.04$ (20 runs averages)
Network GNTM: $N=1000$ $M=2000$

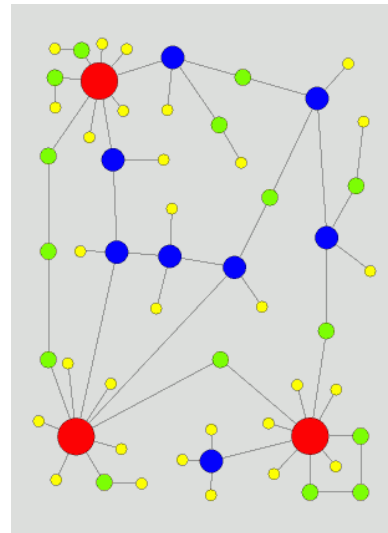
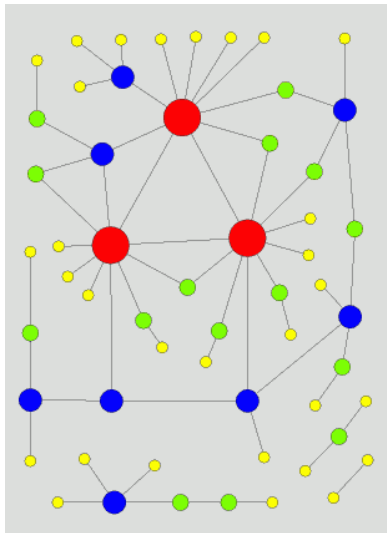
Clustering helps corruption

- In classical epidemics, local clique-clustering slows down the disease spreading because of **re-infection** instead of the infection of healthy
- Here, though, the highly clustered, medium-degree vertices are especially well-suited for the spread of corruption, because a threshold Δ of neighbours has to be corrupt to trigger the α -process

Additive vs. Multiplicative DegreeDegree Correlation



Multiplicative vs. Additive Degree-Correlation



N.B.: identical degree *distribution*:
3 reds (degree 10); 8 blues (degree 4), 15 greens (degree 2), ...

Additive vs. Multiplicative Degree-Degree Correlation

- SF-Networks with multiplicative Degree-Correlation (hierarchical, ...) are more easily corrupted than those with additive Degree-Correlation (polycentric, democratic)
- Especially true for low $\lambda < 3$ (where *very big hubs* exist).

Epidemic Control

- This is an ABSTRACT model!
Only structural & schematic tendencies!
- Positively correlated to corruption:
 α & ε = strength of influence of others
 β = strength of e.g. mass media
- Negatively correlated to corruption:
 Δ =How many neighbours have to be corrupt?
 γ =How strong does the society fight back?
- avoid high clustering
- “Transparency”: $\Delta \uparrow$ $\alpha \downarrow$ $\beta \downarrow$
- “Police”: $\beta \downarrow$ (increase of fear), $\gamma \uparrow$ (uncovering rate)
but $\gamma > \alpha$, β is a “total police state”
- Moral resistance: $\Delta \uparrow$ $\alpha \downarrow$
- (Hierarchical) Decision Systems should be as flat, independent, polycentric as possible!

Knowledge diffusion: first thoughts

- Infection: classical (epsilon) & threshold (alpha)
- no / passive / active knowledge
- activation triggered: local / global / spontaneous
- mean field:
 - the “active” publish
 - there is a time lag
- Forgetting: spontaneous & local effect

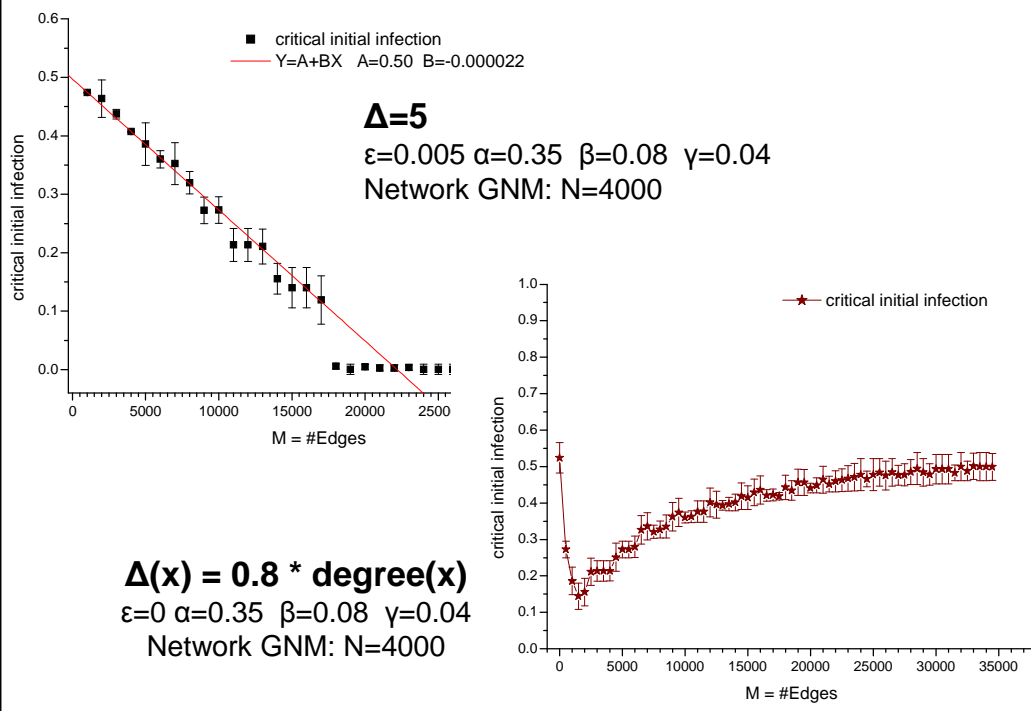
Thank you very much

→ questions?

www.AndreasKrueger.de/networks

→ dissertation/disputation

Absolute vs. Relative Threshold Δ 25 of 22



global observable 26 of 22

$$b = b_t = \frac{1}{N} \sum_{x=1}^N \omega(x) \quad \text{total knowledge prevalence (at time } t)$$

local observables

$$\Omega_t(x) = \sum_{x \sim y} \omega(y) \quad \text{number of knowing neighbours of } x = 3$$

$$\Phi_t(x) = \sum_{x \sim y} \frac{1}{d(y)} \omega(y) \quad \text{local knowledge inflow} = 1 + 1/3 + 1/6 = 1.5$$

Inner structure of projects is *not* FullGraph, but now we account for that:

1/degree weighing of the knowing neighbours

