

# Modelling Knowledge

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**NEMO-Workshop**

- Part 1:  
Generalized Epidemic Process (GEP)  
for Knowledge Diffusion („Spreading“)
- Part 2: for further modelling:  
Knowledge about *Knowledge*...
- Part 3:  
Shared/Complementary Knowledge Model  
for Partner Choice

# Part 1: Diffusion of knowledge

Generalized Epidemic Process (GEP):

- classical epidemics
- threshold epidemics
- mean-field effect
- forgetting, i.e. active / passive knowledge
- Initial infection:  
„seed group“ of interconnected nodes

Bimodal Network with  $N = N_{orgs} + N_{projs}$  nodes

$x \sim y \iff$  **edge** between  $x$  and  $y$

$d(x)$  = degree of  $x$   
= number of projects in which **organisation**  $x$  participates  
*or* number of participating organisations in **project**  $x$

$$\omega(x) = \omega_t(x) = \begin{cases} 0 & \text{unaware} \\ 1 & \text{knowing} \end{cases} \text{ at time } t$$



first model:  
*one* type of knowledge



project  $x$  can be unaware/knowing and  
organisation  $x$  can be unaware/knowing

first model:  
*no* distinction between  
organisations & projects

## global observable

$$b = b_t = \frac{1}{N} \sum_{x=1}^N \omega(x) \quad \text{total knowledge prevalence (at time } t)$$

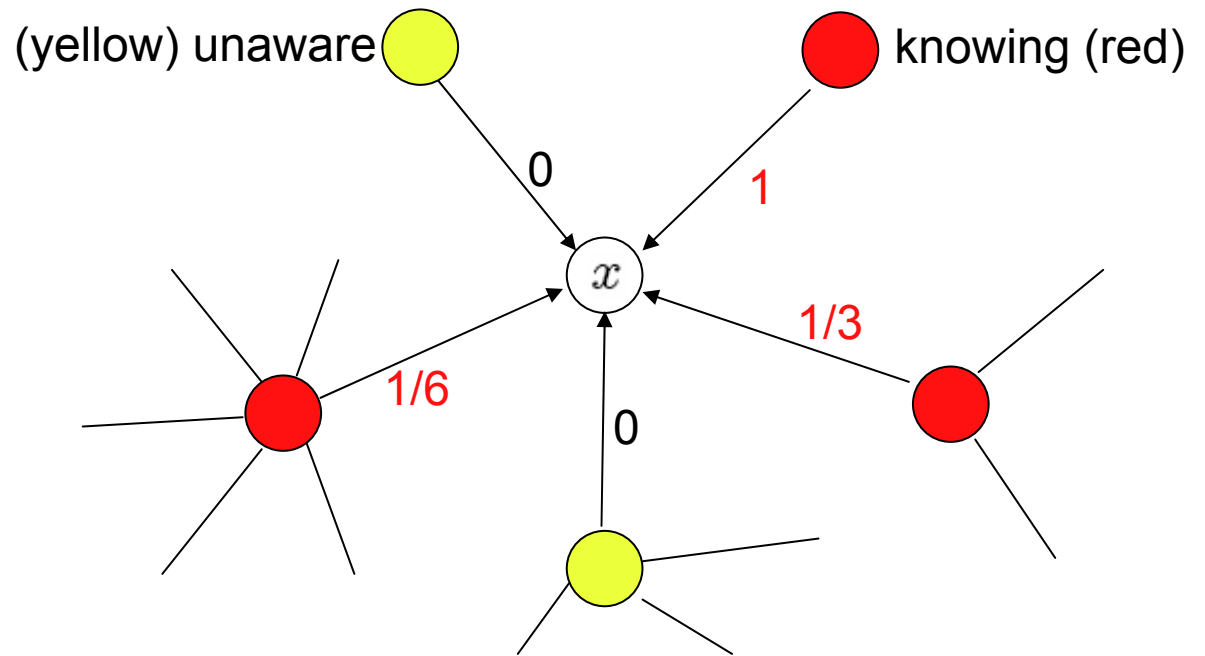
## local observables

$$\Omega_t(x) = \sum_{x \sim y} \omega(y) \quad \text{number of knowing neighbours of } x = 3$$

$$\Phi_t(x) = \sum_{x \sim y} \frac{1}{d(y)} \omega(y) \quad \text{local knowledge inflow} = 1 + 1/3 + 1/6 = 1.5$$

Inner structure of projects is *not* FullGraph, but now we account for that:

1/degree weighing of the knowing neighbours



# ( $\epsilon$ ) epsilon-process ~classical epidemics

- Local infection by knowing neighbours
- The epsilon-process has a very low probability  $\epsilon$ , but:
- The more neighbours knowing, the higher the probability to get knowing:

$$P_{0 \rightarrow 1} = \epsilon \cdot \Phi(x)$$

$$\Phi_t(x) = \sum_{x \sim y} \frac{1}{d(y)} \omega(y)$$

But this rather weak epsilon process only happens below a threshold ...  $1 \leq \Omega(x) < \Delta$

# ( $\alpha$ ) alpha-process: delta-threshold infection

If the *number of knowing neighbours*

exceeds a threshold  $\Delta$   $\Omega(x) \geq \Delta$

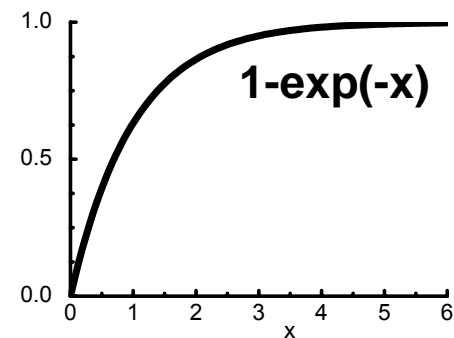
suddenly there is a *higher* probability  $\alpha$  to get knowing

$$\mathbf{P}_{0 \rightarrow 1} = \alpha \cdot \left(1 - e^{-\Phi(x)}\right)$$

Degree weighed inflow

$$\Phi_t(x) = \sum_{x \sim y} \frac{1}{d(y)} \omega(y) \quad \text{shift into } [0 \dots 1]:$$

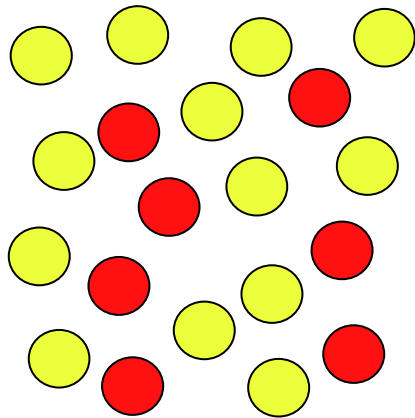
$$(1 - e^{-\Phi_t(x)}) = \begin{cases} \sim 0 & \text{for } \Phi_t(x) \text{ small} \\ \sim 1 & \text{for } \Phi_t(x) \text{ large} \end{cases}$$



# ( $\beta$ ) beta-process: mean-field influence infects

- i.e. mass media, intuition about the state of the whole system, journals, ... = „mean-field“.
- Proportional to square of relative prevalence

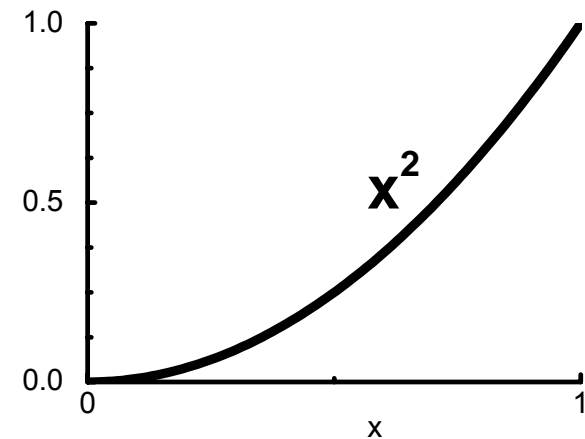
$$P_{0 \rightarrow 1} = \beta (b_t)^2$$



$$b_t = \frac{1}{N} \sum_{x=1}^N \omega(x)$$

total knowledge prevalence

$$7 / 20 = 0.35$$





# ( $\gamma$ ) gamma-process: forgetting passive knowledge

- The less-knowing my neighbours,  
the higher my  $\gamma$ -process-forgetting

$$\mathbf{P}_{1 \rightarrow 0} = \gamma \left( 1 - \frac{\overbrace{\Omega_t(x)}^{\text{Ratio of unaware neighbours}}}{\underbrace{d(x)}_{\text{Ratio of knowing neighbours}}} \right)$$

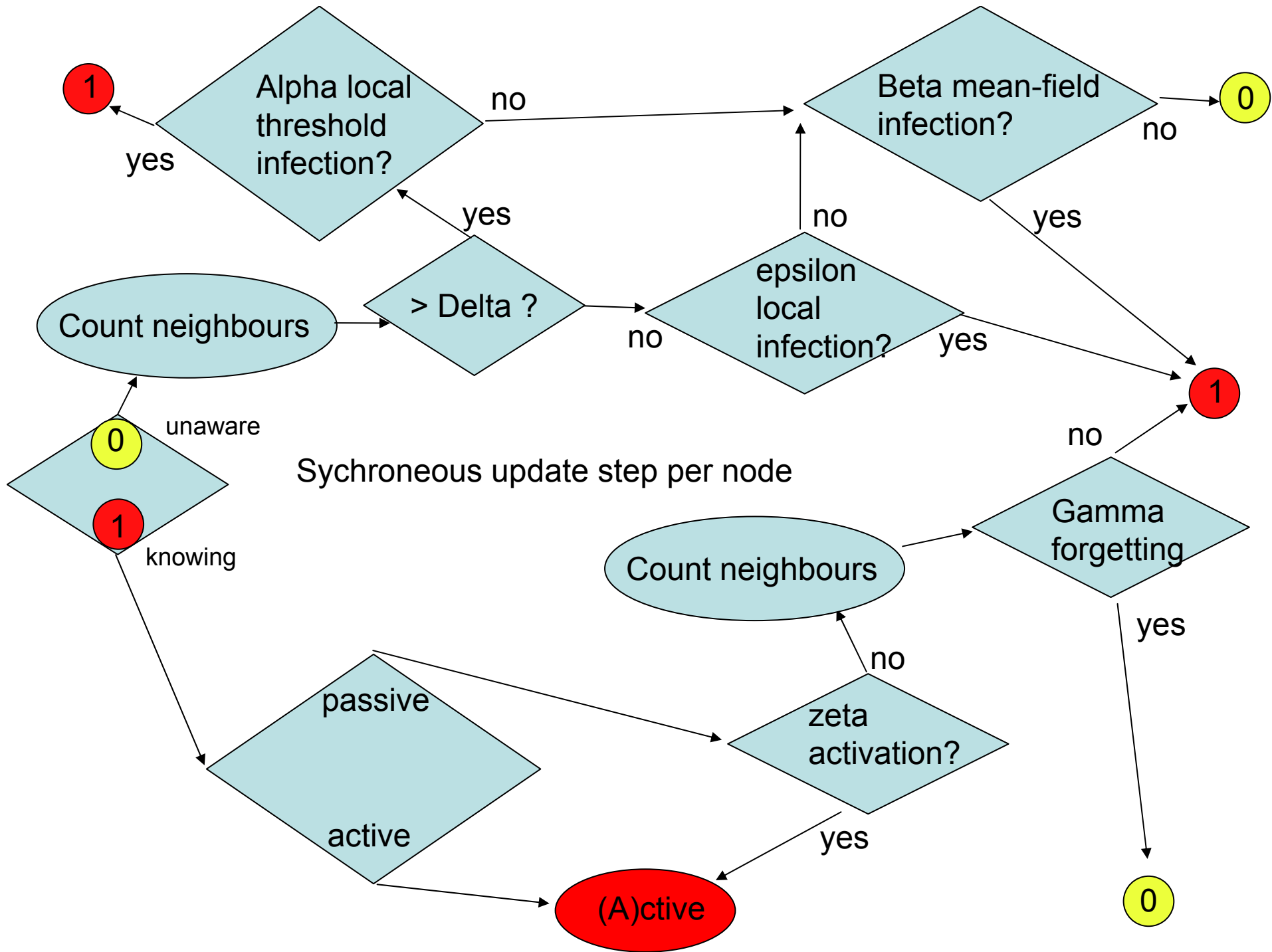
But I can only forget PASSIVE knowledge. ACTIVE knowledge stays with me...

# ( $\zeta$ ) zeta-process: activation of passive knowledge

- Each time step there is a (constant) probability  $\zeta$  to get from „passive“ to „active“ knowledge

$$P_{1 \rightarrow A} = \zeta$$

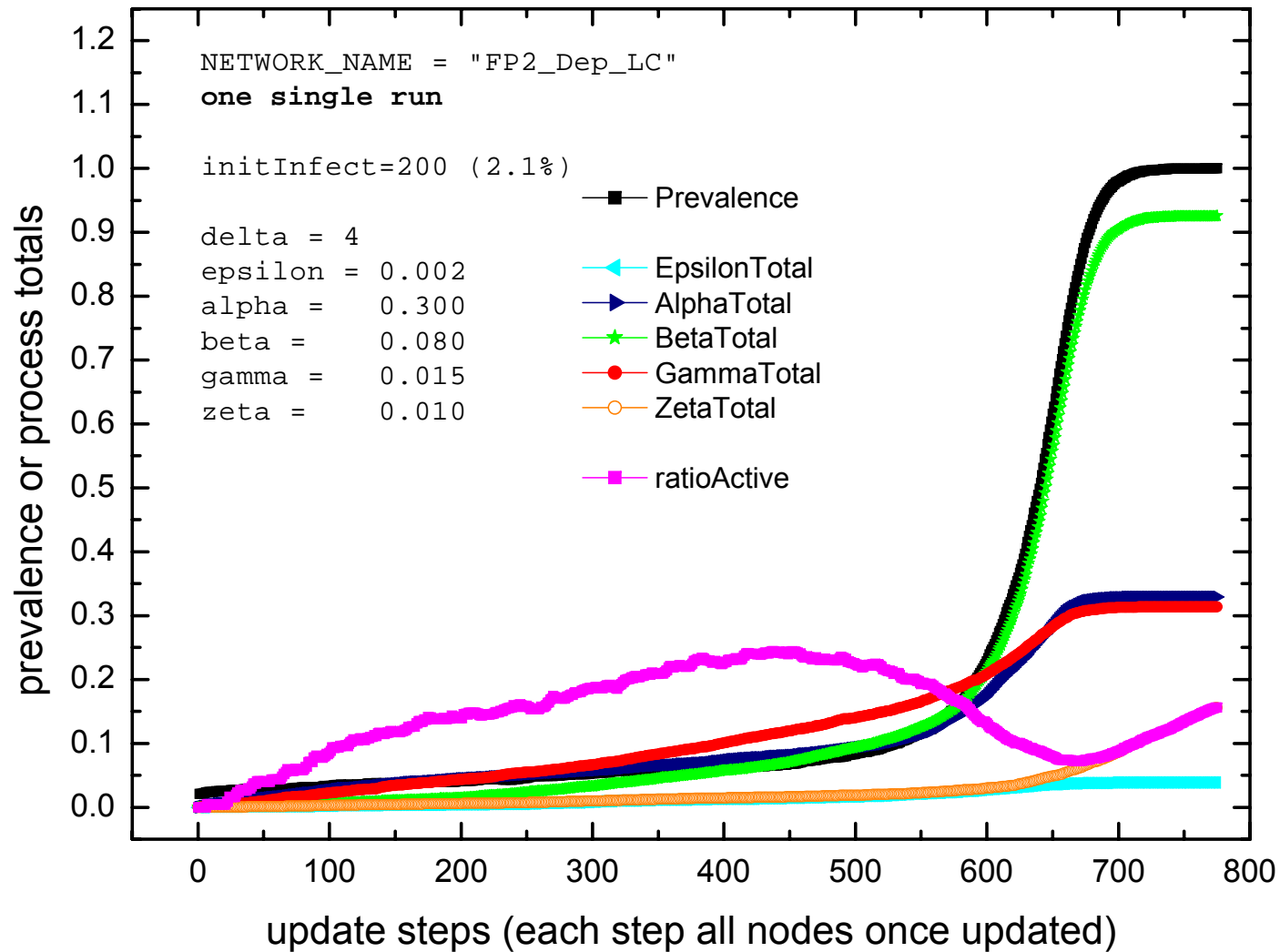
- Only passive knowledge can be forgotten. Once activated, the node stays knowing forever.
- Possible extension: Active knowledge „counts“ more than passive knowledge (e.g.  $A=3$ )



**GEP runs**

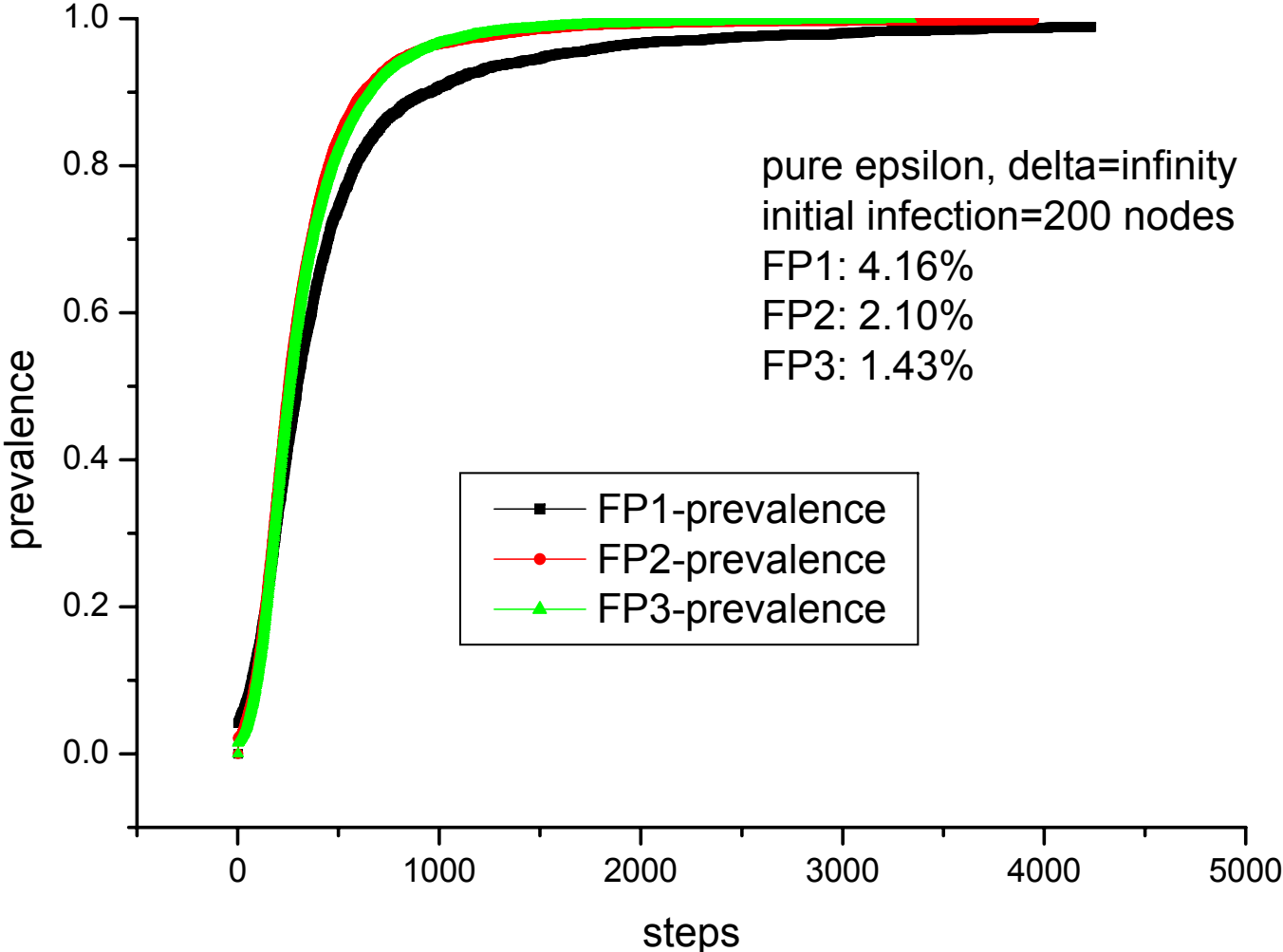
# One infection run, on FP2

initial infection: 200 nodes

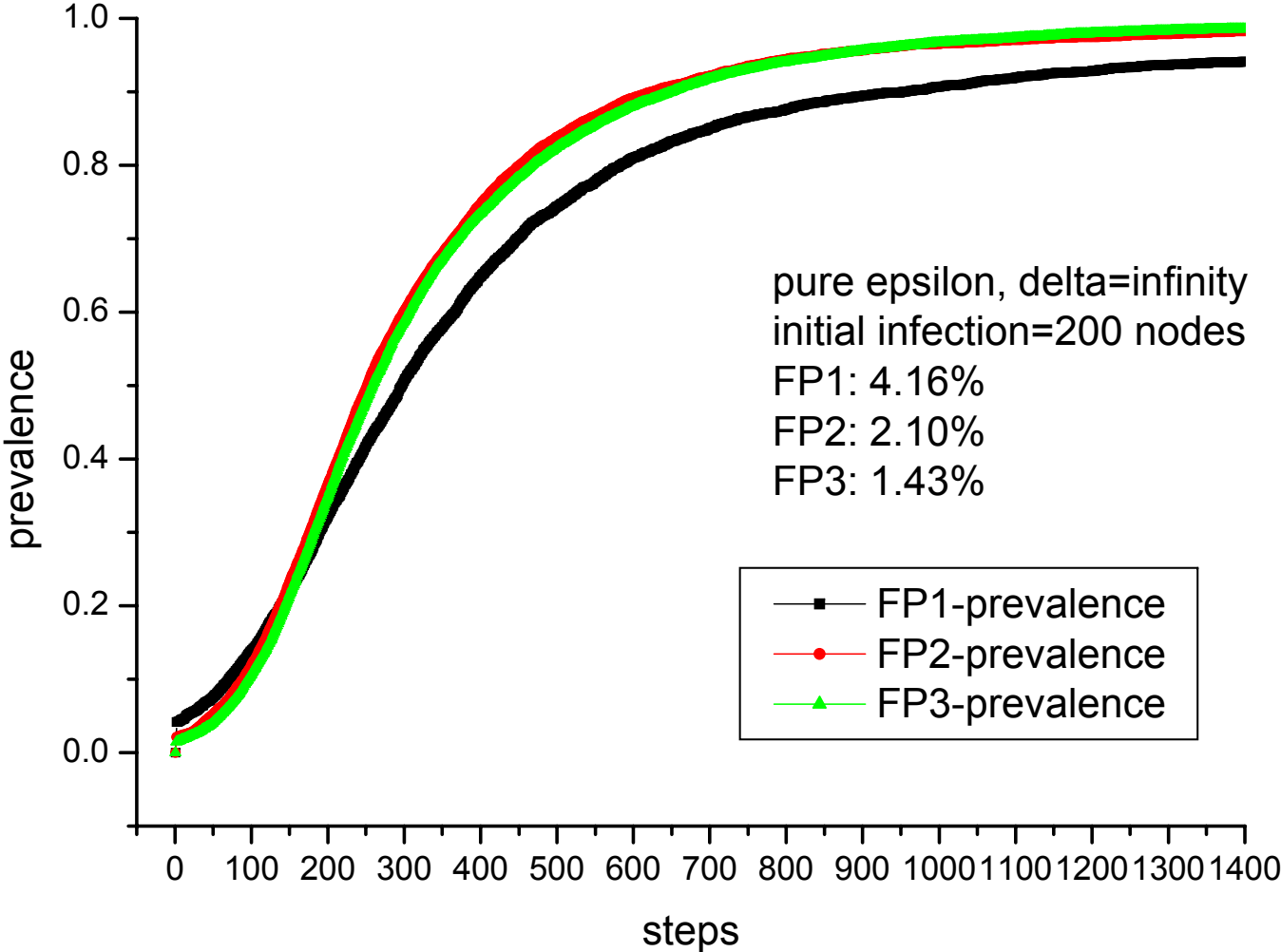


# Pure Epsilon process $\epsilon=0.04$

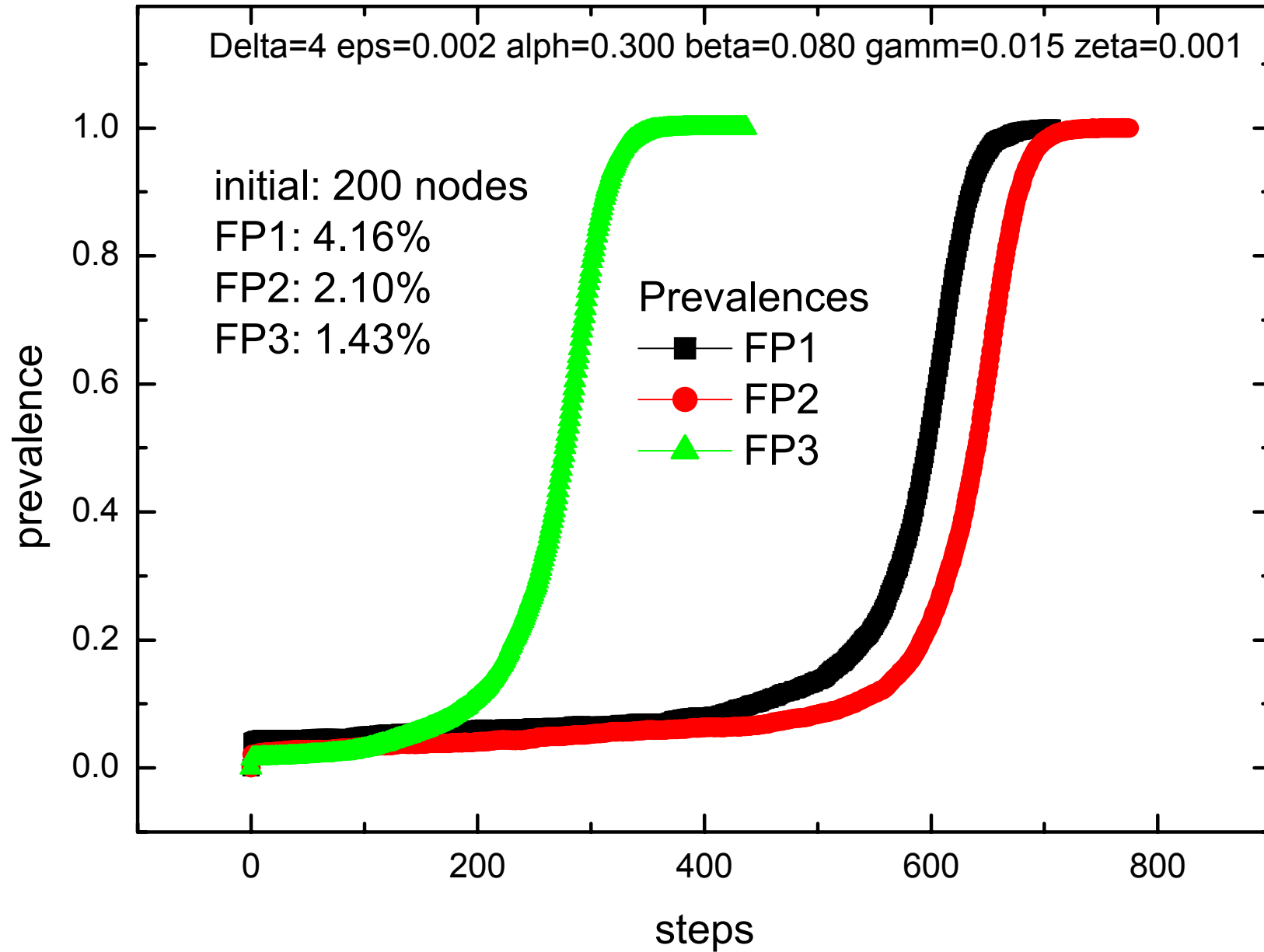
$\alpha=0$   $\beta=0$   $\gamma=0$   $\zeta=0$   $\delta=\infty$



# Pure Epsilon process $\epsilon=0.04$ $\alpha=0$ $\beta=0$ $\gamma=0$ $\zeta=0$ $\delta=\infty$

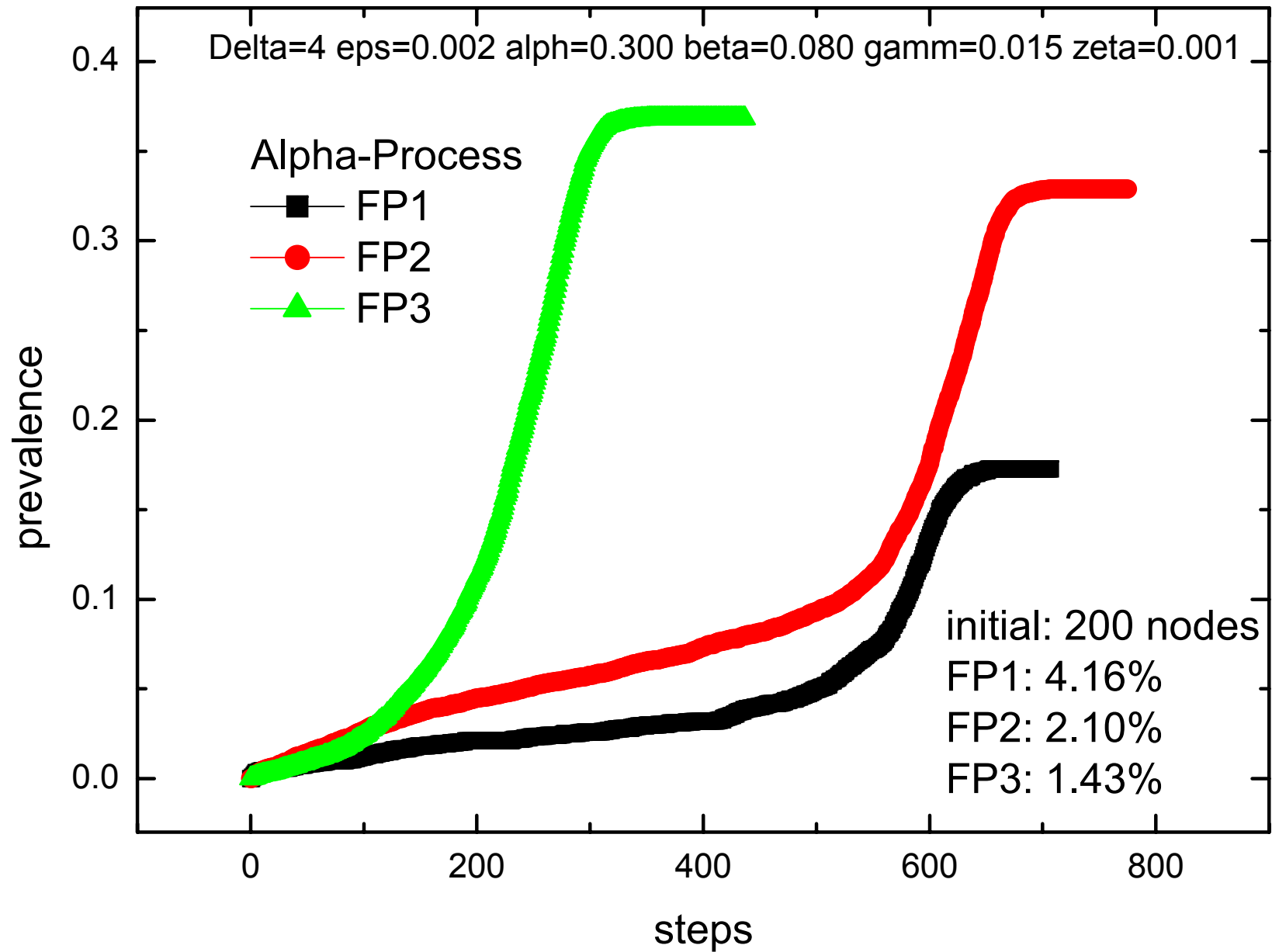


One infection run: **Prevalence** FP1 FP2 FP3 - comparison





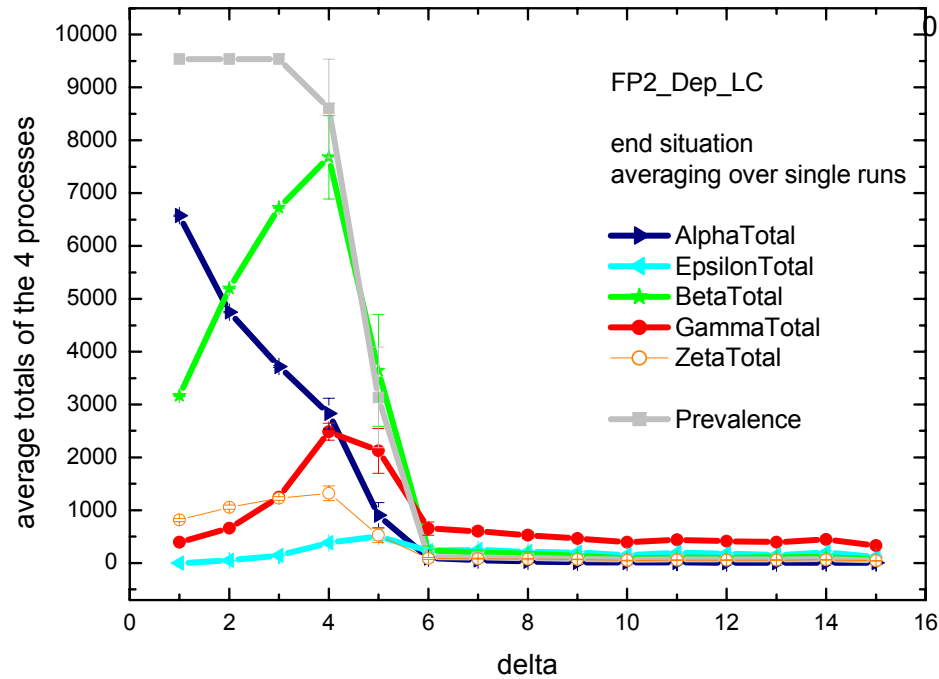
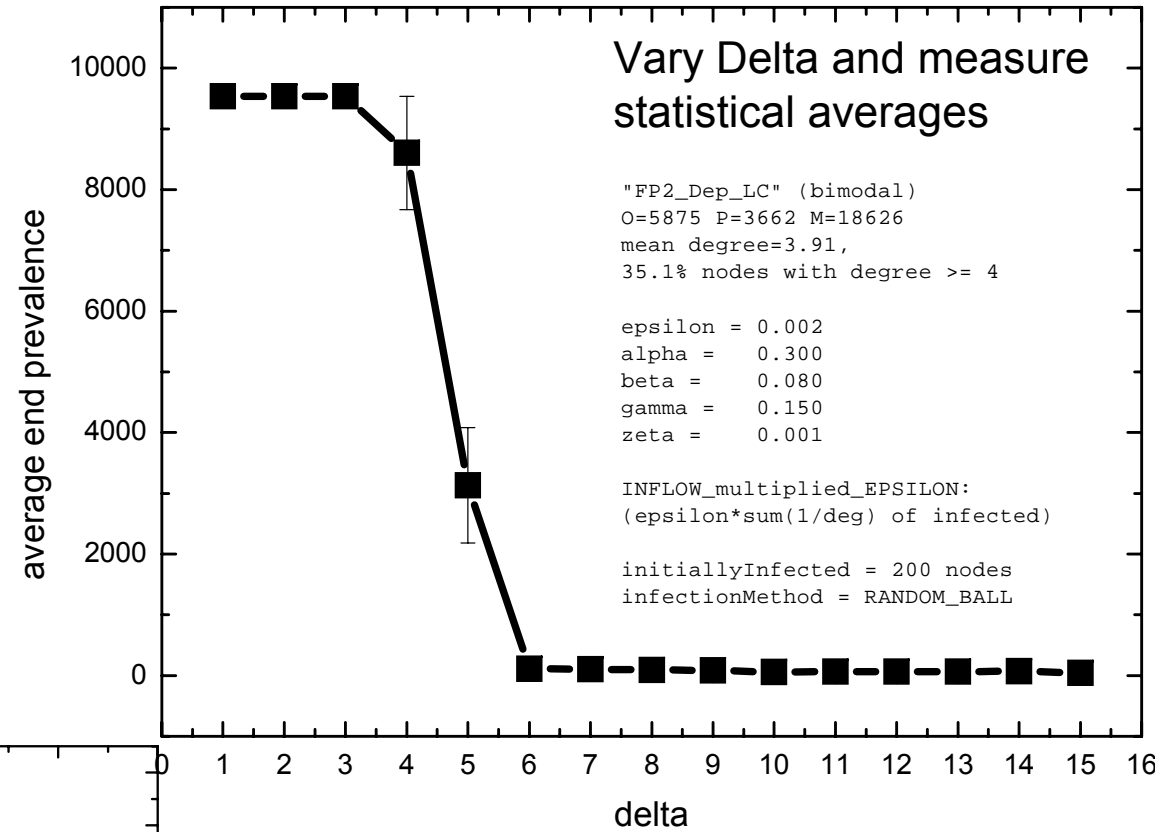
One infection run: **Alpha-Process** FP1 FP2 FP3 - comparison



# FP2

Variation of delta

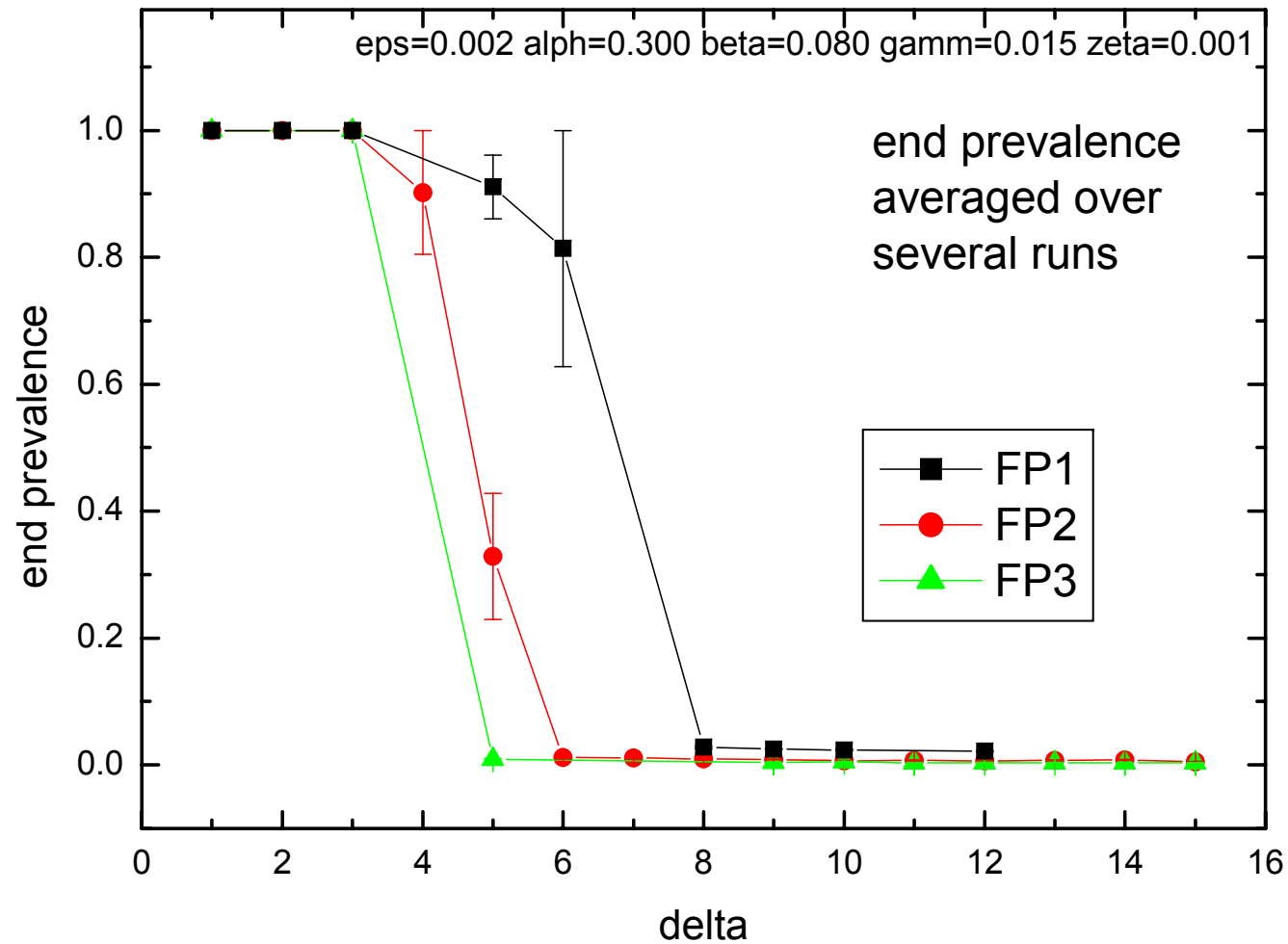
Many runs,  
averages of **end results**



# FP1, 2, 3

# Variation of delta-threshold

Many runs, averages of **end** results



## Planned *next* extensions:

- infectious time is only short after infection
- competing knowledge types:
  - first steps into high-dimensional knowledge representation
  - no *active* knowledge of all types possible
  - Majority rules for local and mean-field processes

# Part 2: stylized facts about knowledge..



Project no. NEST-2006-028875

NEMO

Network Models, Governance and R&D Collaboration Networks

Instrument: Specific Targeted Research Project (STREP)

Thematic Priority: NEST-Adventure

## Deliverable D1.1

### Conceptual and empirical foundations of R&D network dynamics

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Petra Ahrweiler  
Michael Barber  
Andreas Brandes  
Barbara Heller-Schuh  
Nicolas Jonard  
Magnus Kutz  
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Petra Wagner-Luptacik  
Matthias Weber

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Project co-funded by the European Commission within the Sixth Framework Programme (2002-2006)		
Dissemination Level		
PU	Public	
PP	Restricted to other programme participants (including the Commission Services)	
RE	Restricted to a group specified by the consortium (including the Commission Services)	
CO	Confidential, only for members of the consortium (including the Commission Services)	X

# Knowledge: 3 fields of application in NEMO

- **knowledge diffusion** = i.e. "spreading" of innovation
  - in an *existing* network - **how does knowledge spread?**
  - example: the term "**chaos**" (for nonlinear systems) has spread all over science
  - our GEP model addresses knowledge spreading (Part 1)
- projects: **partner choice** by their knowledge tuple
  - for project goal *and* for self-interest
  - choice of best match among network partners – locally ...
  - ... and global search
- **network variation**
  - after a (successful) project
  - strengthen and weaken some bonds
  - change the network accordingly

# Knowledge Types

- active versus passive knowledge
  - no forgetting of active knowledge
  - usage activates knowledge
  - active knowledge likely to have stronger infection effect
- Knowledge  $K_i$  is a tuple in high-dimensional *metric* space

# Projects: *choice of partners*

- for mutual understanding  
shared knowledge necessary
- incentive for cooperation is  
complementary knowledge
- *project interest versus all self-interests*
- secret and non-transferable knowledge, patents
- trust ~ history
  - relational embeddedness: (positive) past experiences
  - structural embeddedness: indirect ties
- project-coordinators are  
more likely to be chosen again



# Choice of partners: Heterogeneity!

- local ties vs. cluster-spanning
  - banks experienced with *cluster-spanning ties* are more likely to establish cluster-spanning ties in the future
  - ... versus *within-clique-ties*
- more vs selective
  - organisations with *few* contacts tend to *add more* partners
  - organisations with *many* contacts are likely to be *selective*

# Project

- project knowledge stock
  - "sum" of participants
  - sum over knowledge-*subspace* only!
- during cooperation
  - generation of completely new knowledge
  - activation of passive knowledge
  - knowledge exchange, by imitation
  - directed versus mutual learning
- project goal
  - project *goal*: certain knowledge tuple
  - project *outcome*: certain knowledge tuple
  - *products* can be produced
    - once the necessary knowledge tuple exists

# Project

- project proposal
  - Project goal is evaluated
  - Project *goal in proposal* might be different ...
  - ... from project *outcome that can be produced* from knowledge stock!
  - proposal generation phase is knowledge exchange already (even if project is not contracted)
- inner structure of project
  - 4-10 work packages
    - cooperation is mainly done in workpackages
    - intra project structure 11%-99% density of FullGraph
  - overlay of management/coordination network: star network or star-of-cliques

# Knowledge outside projects

- organisations bring *initial knowledge*
- working outside cooperation  
*also* increases knowledge
- mobility of knowledge workers  
across firms

# Limits of the individual (organisation)

- absorptive capacities:  
per timestep limited learning
- heterogeneity of individuals "intelligence,"
- no one knows everything
- misunderstanding
- forgetting

# „subjectively meaningful“

- context-embedded knowledge
- Individual knowledge stock  
determines what *can* be learnt
- received knowledge less, different  
or other than sended knowledge
- from the outside, systems cannot be  
"informed" with a certain and sure  
knowledge transfer, but rather activated  
to learn themselves from given offers

Part 3:

multidimensional knowledge  
& partner choice

$$x=(x_1,\dots,x_n)$$

$$y=(y_1,\dots,y_n)$$

$$|x_i - y_i| \geq \delta_{comp}$$

$$|x_i - y_i| \leq \delta_{shared}$$

### Complementary Knowledge Subspace

$$D_{comp} = \{i : |x_i - y_i| \geq \delta_{comp}\}$$

$$d_{comp} = \#D_{comp}$$

complementary knowledge number

### Shared Knowledge Subspace

$$D_{shared} = \{i : |x_i - y_i| \leq \delta_{shared}\}$$

$$d_{shared} = \#D_{shared}$$

shared knowledge number

The **probability**  $P(x \sim y)$  that  $x$  cooperates with  $y$  and produces new knowledge is a monotone rising function of the size of common speech, measured by  $d_{shared}$ :

$$P(x \sim y) \propto d_{shared}$$

The **value** of this new knowledge component is a monotone rising function of the number of topics in which the cooperators complement each other. This number is measured by  $d_{comp}$ :

$$x_{n+1} \propto d_{comp}$$



Thank you.

# Additional Slides

# Knowledge representation

As this is an interdisciplinary workshop, example first 😊

Given 4 knowledge tuples  $K_1, K_2, K_3, K_4$

$$\begin{array}{l} K_1 = (1, 1, 0\dots 0) \\ K_2 = (1, 1, 0\dots 0) \\ K_3 = (0, 1, 0\dots 0) \\ K_4 = (0, 0, 1, 0\dots 0) \end{array} \begin{array}{l} \text{)} \\ \text{)} \\ \text{)} \\ \text{)} \end{array}$$

Let us define a *distance function*  $d$  with this behaviour:

- $d(K_1, K_2) = 0$   
knowledge  $K_1$  and  $K_2$  are *identical*
- $d(K_1, K_3) < d(K_1, K_4)$  both compared to  $K_1$ ,  
knowledge  $K_3$  is *more similar* than  $K_4$

# Knowledge representation: Metric space

A *metric space*  $(M, d)$  are a set  $M$   
and a *distance* function  $d : M \times M \rightarrow \mathbb{R}$ .

The function  $d$  can **compare** elements of  $M$  and fulfills 3 conditions:

- (1) *identity of indiscernibles*     $d(x, y) = 0$  only if  $x = y$
- (2) *symmetry*     $d(x, y) = d(y, x)$
- (3) *triangle inequality*     $d(x, y) + d(y, z) \geq d(x, z)$

these three conditions combine to the property of

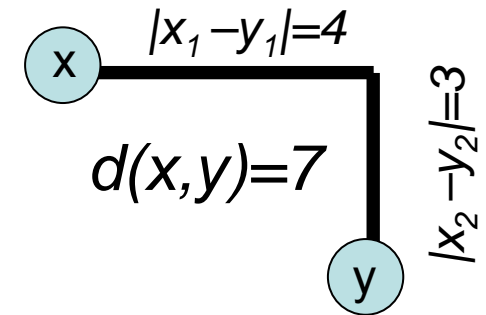
- (4) *non-negativity*     $d(x, y) \geq 0$

Let the *knowledge space*  $\mathfrak{K} = \mathbb{R}^n$  be an  $n$ -dimensional metric space, over the real numbers  $\mathbb{R}$ . A knowledge tuple  $K \in \mathfrak{K}$  can then be written as real coefficients  $K = (k_1, k_2, \dots, k_n)$  with  $k_i \in \mathbb{R}$

Any function with conditions 1-4 (e.g. the Euclidian distance) can serve as a metric. Example: the **Manhattan distance** sums differences in all dimensions:

$$d(x, y) = \sum_{i=1}^n |x_i - y_i|$$

$$= |x_1 - y_1| + |x_2 - y_2| + \dots + |x_n - y_n|$$



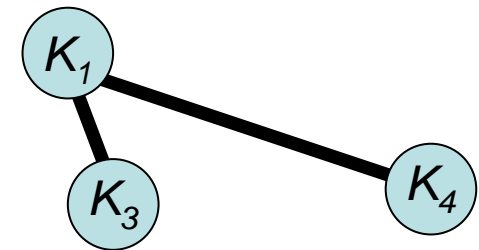
Given 4 knowledge tuples  $K_1, K_2, K_3, K_4$

$$K_1 = (1, 1, 0...0)$$

$$K_2 = (1, 1, 0...0)$$

$$K_3 = (0, 1, 0...0)$$

$$K_4 = (0, 0, 1, 0..0)$$



Metric spaces are about DISTANCES

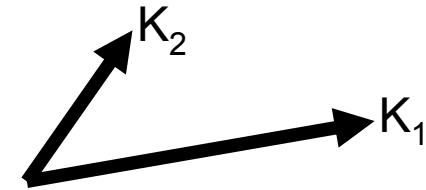
- $d(K_1, K_2) = 0 = |1 - 1| + |1 - 1| + |0 - 0| + \dots$
- $d(K_1, K_3) = 1 = |1 - 0| + |1 - 1| + |0 - 0| + \dots$
- $d(K_1, K_4) = 3 = |1 - 0| + |1 - 0| + |0 - 1| + |0 - 0| + \dots$

in contrast to *metric* space: **Vector space**

"within NEMO, an agreement has been made to represent knowledge pragmatically as **vectors** in multidimensional space"

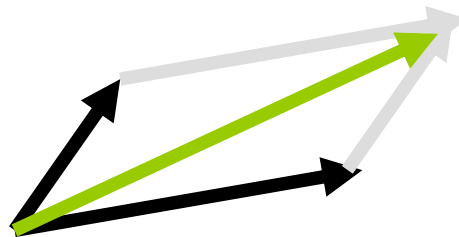
(Deliverable D1.1 p. 43)

→ ???

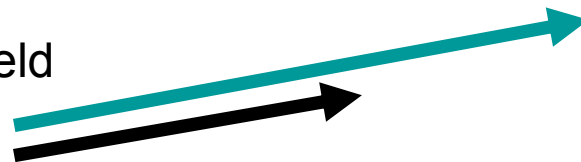


## Some properties of Vector Spaces

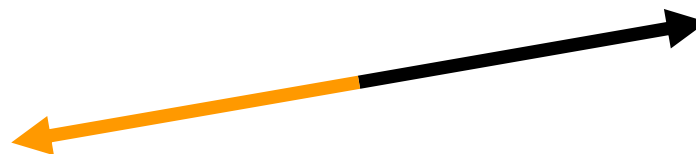
Vector addition  
(parallelogram rule)



Vector multiplied by field  
element („scaling“)



Vector addition has  
inverse elements



For vector  $v$  exists vector  $w$  so that  $v+w=0$

→ ???

*All* simulation results up to now

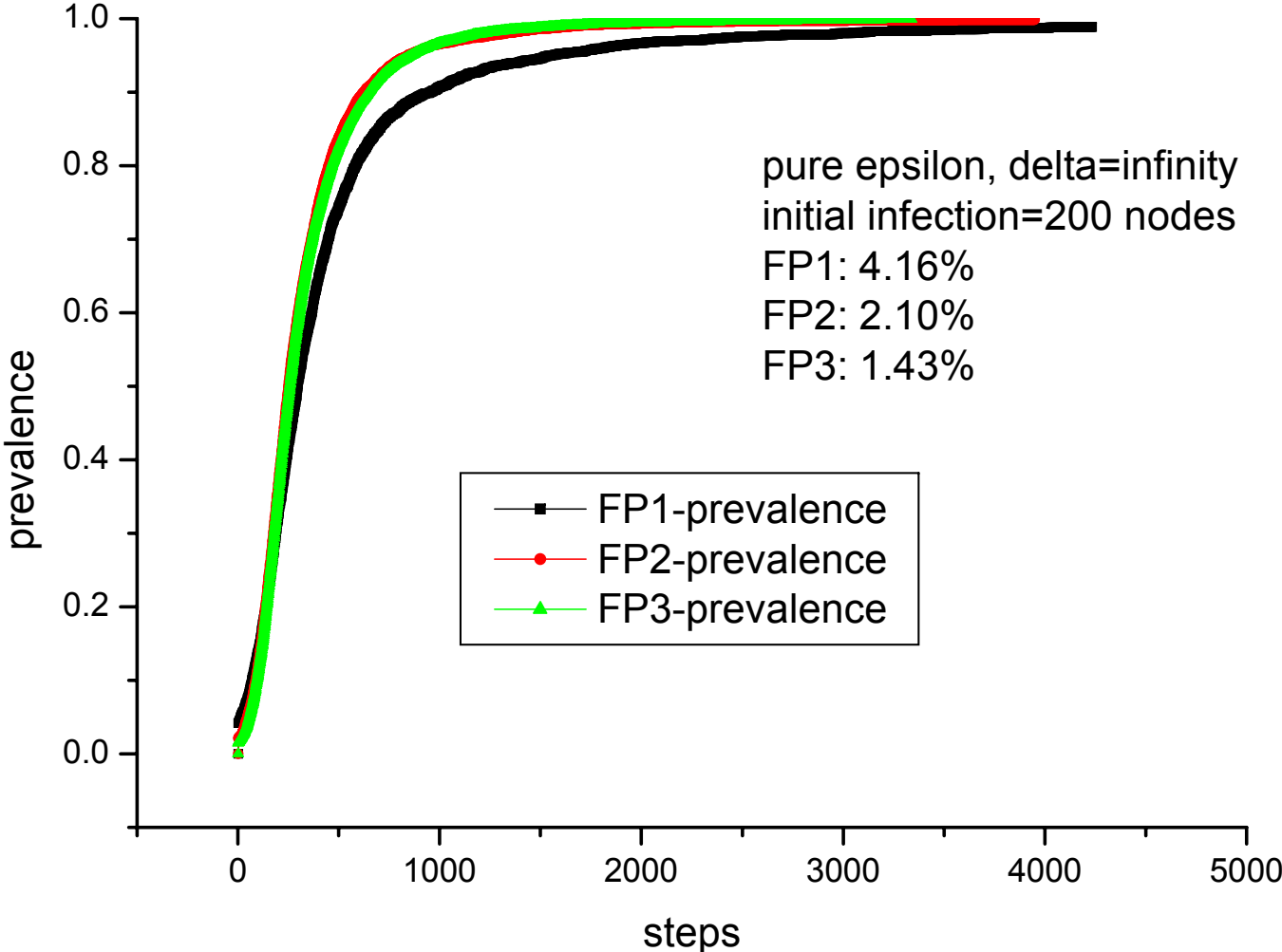
(before was a selection only)

Pure epsilon process  
classical epidemics +  $1/\text{degree-effect}$

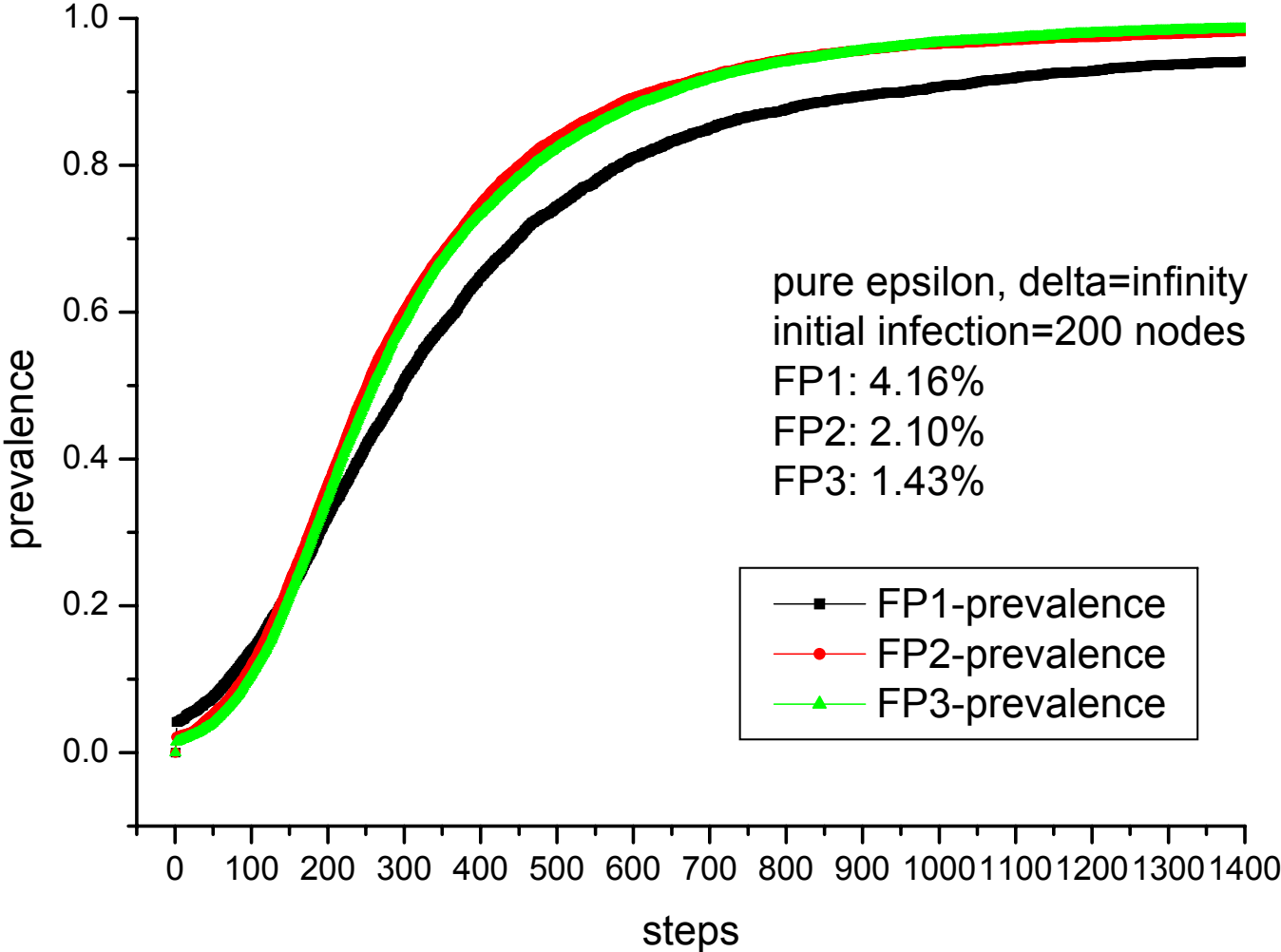


# Pure Epsilon process $\epsilon=0.04$

$\alpha=0$   $\beta=0$   $\gamma=0$   $\zeta=0$   $\delta=\infty$

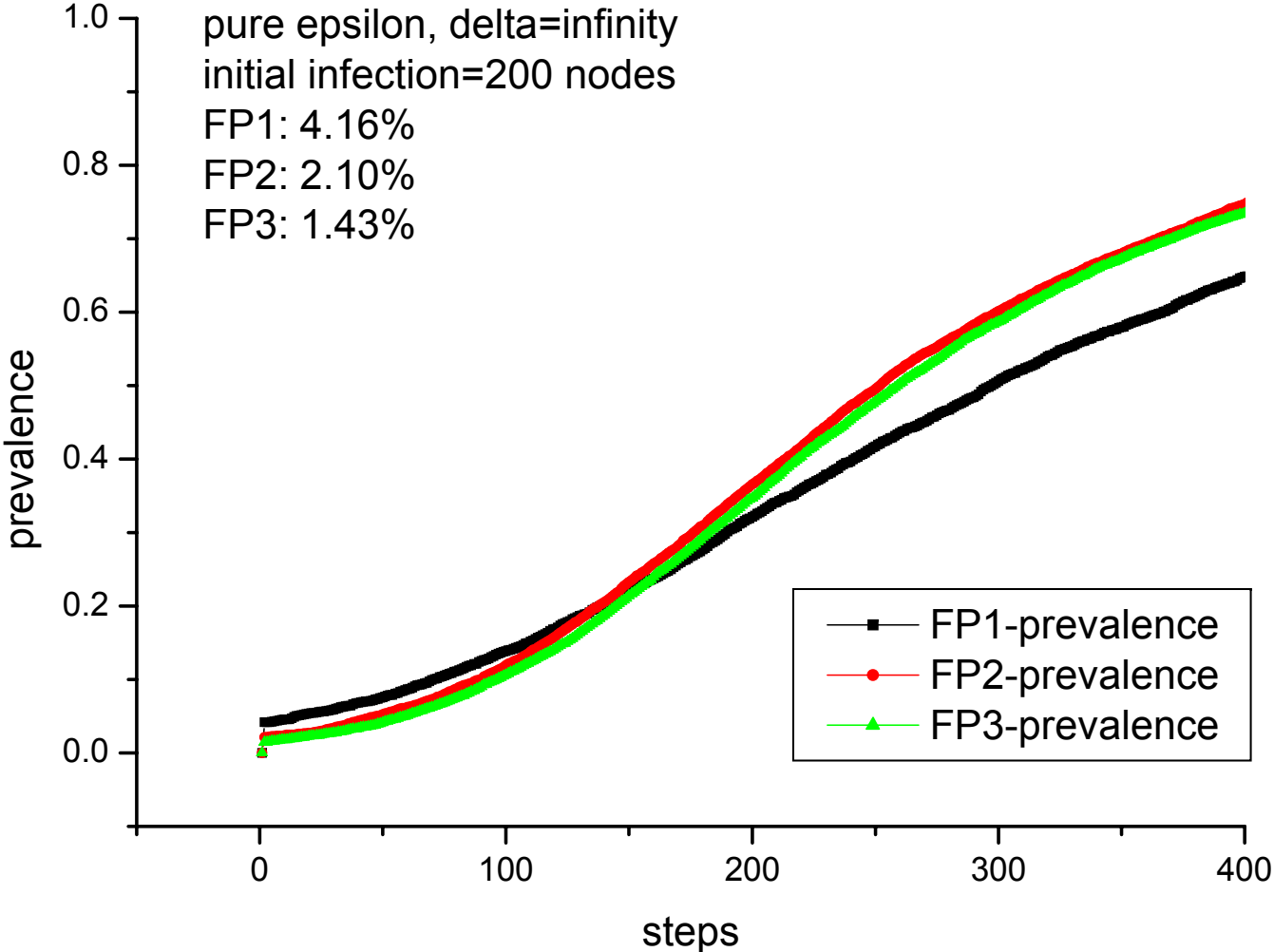


# Pure Epsilon process $\epsilon=0.04$ $\alpha=0$ $\beta=0$ $\gamma=0$ $\zeta=0$ $\delta=\infty$



# Pure Epsilon process $\epsilon=0.04$

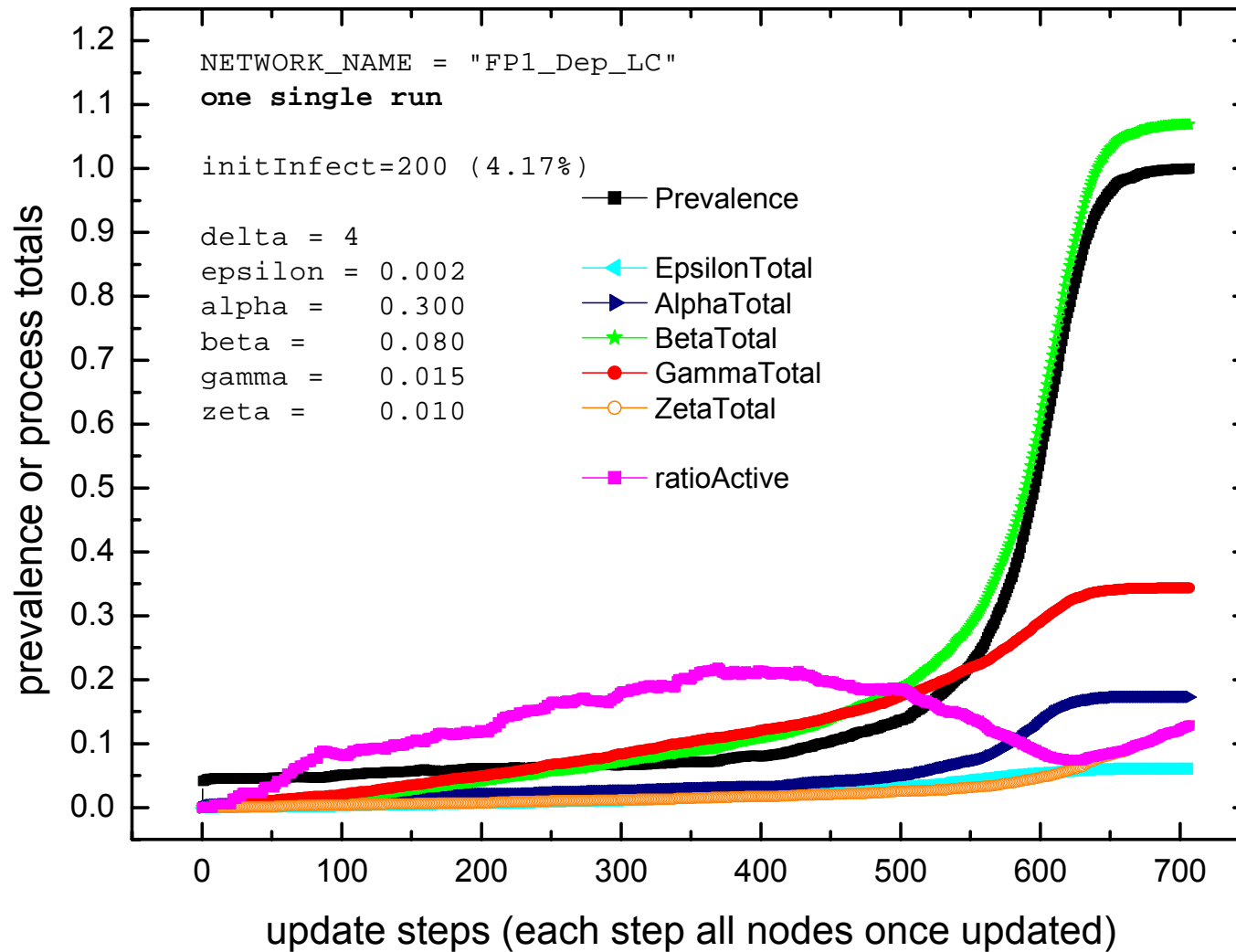
$\alpha=0$   $\beta=0$   $\gamma=0$   $\zeta=0$   $\delta=\infty$



all processes switched on

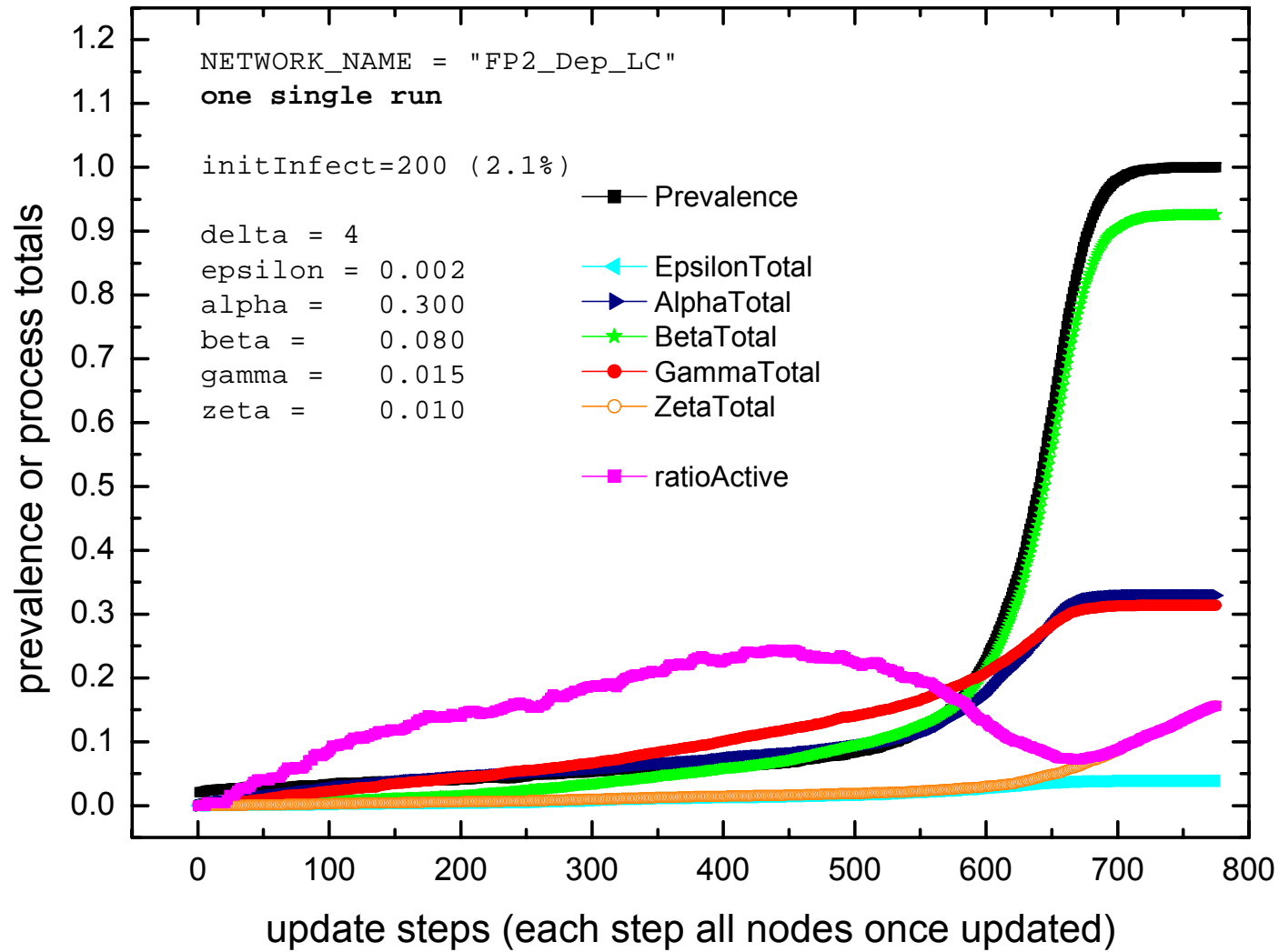
# One infection run, on FP1

initial infection: 200 nodes



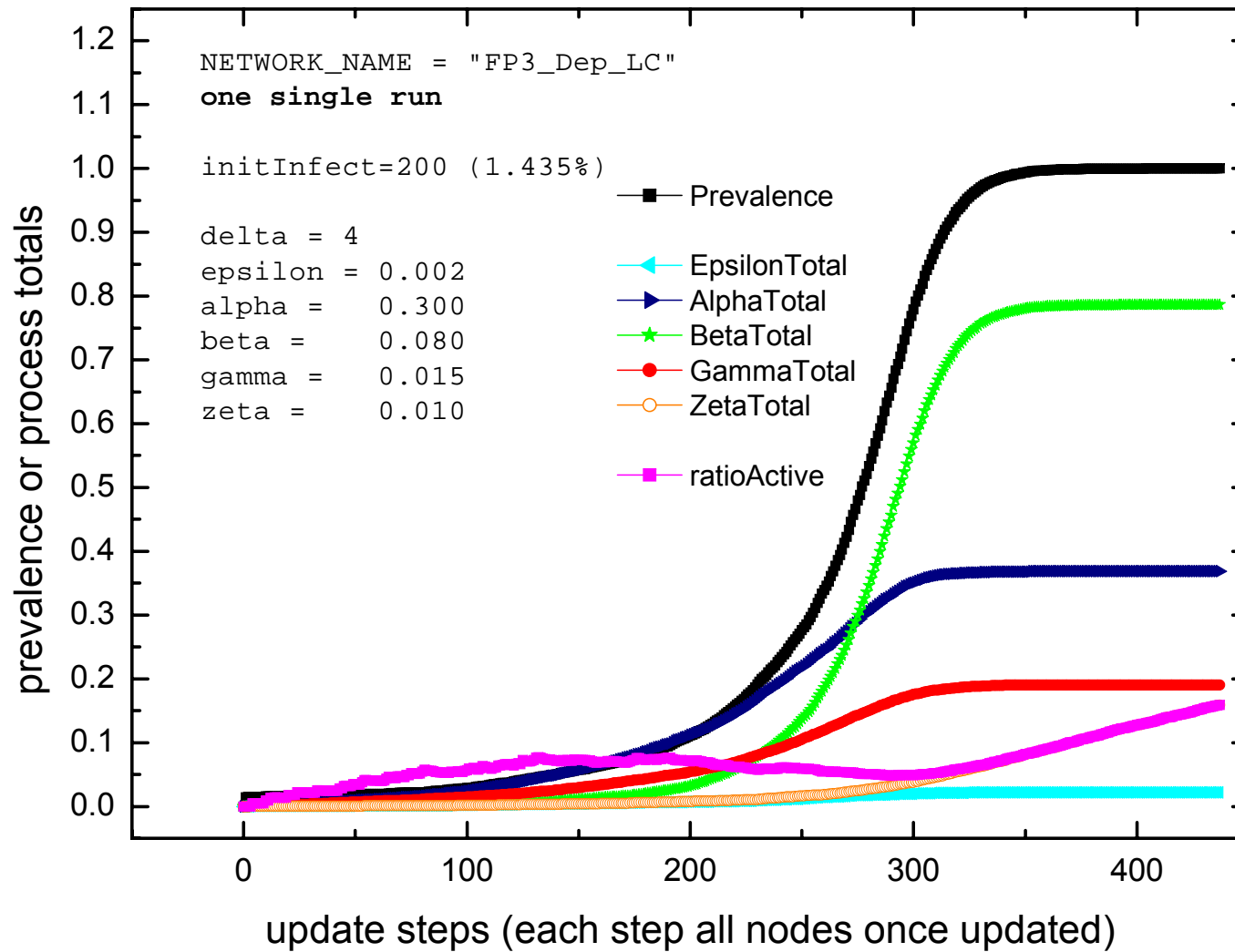
# One infection run, on FP2

initial infection: 200 nodes



# One infection run, on FP3

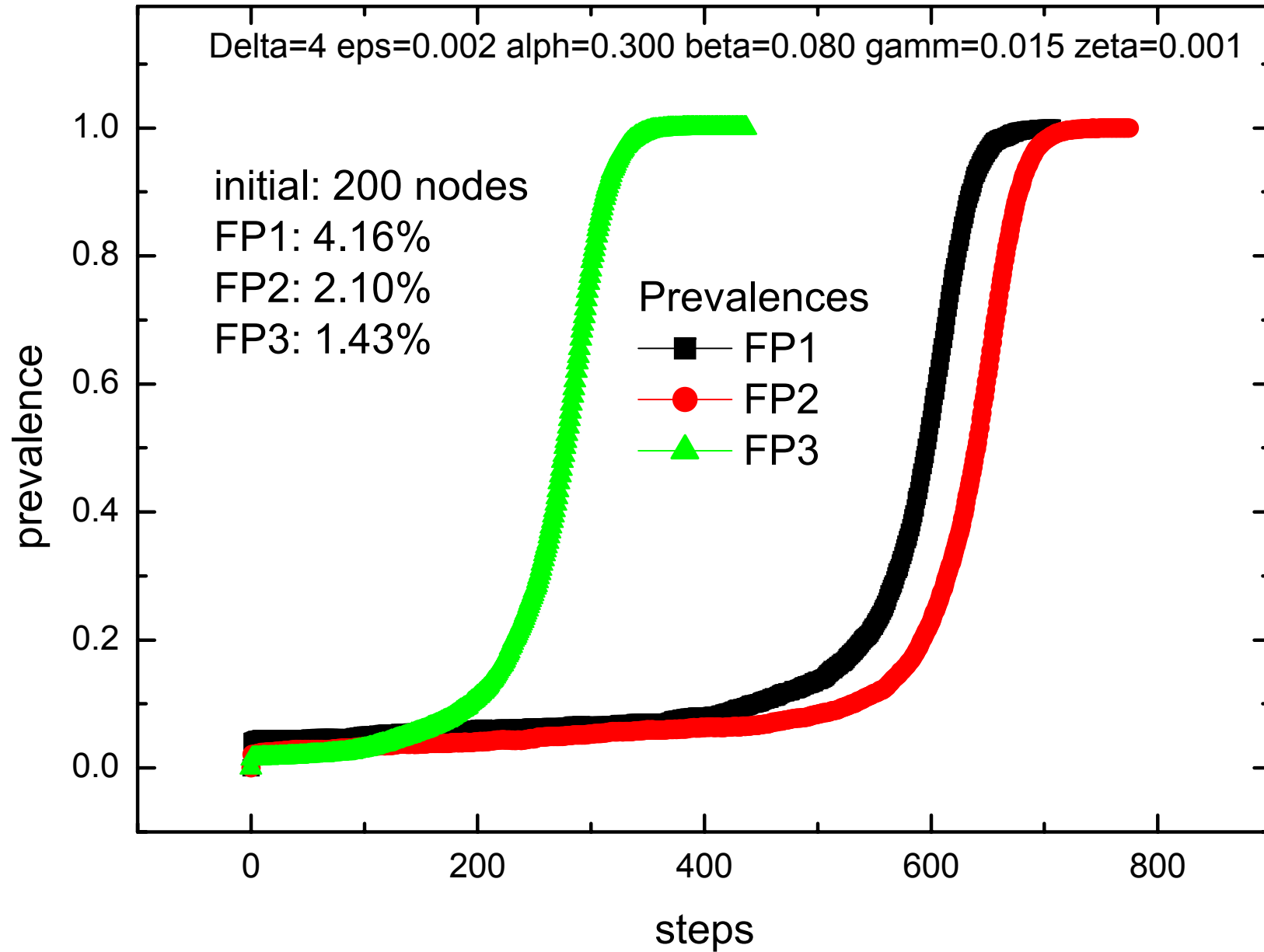
initial infection: 200 nodes



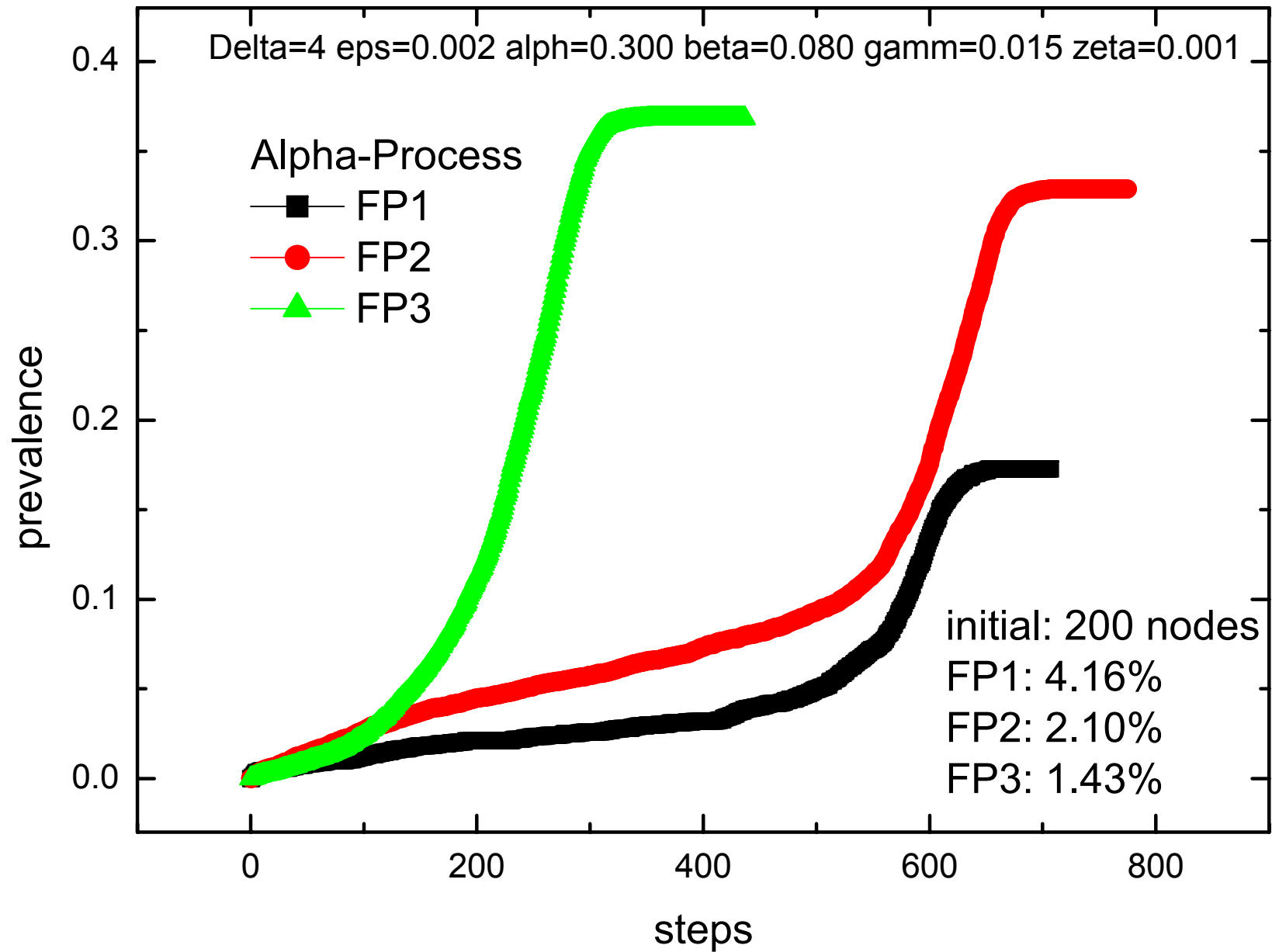
# Comparison of FP 1 2 3



One infection run: Prevalence FP1 FP2 FP3 - comparison



One infection run: **Alpha-Process** FP1 FP2 FP3 - comparison



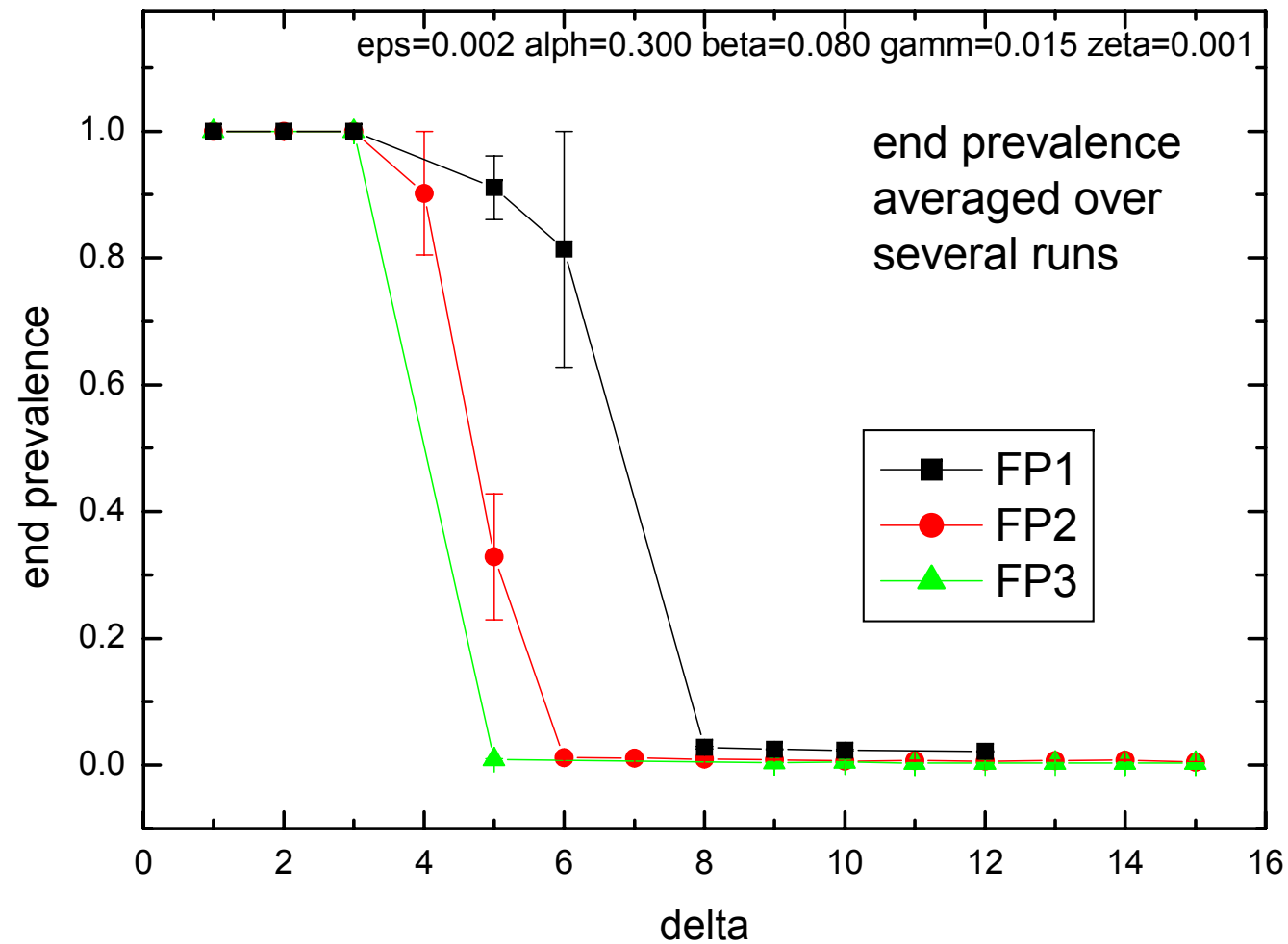
# delta sweep

- Variation of delta
  - Small delta = „big news“
  - Big delta: I need to hear it many times

# FP1, 2, 3

# Variation of delta-threshold

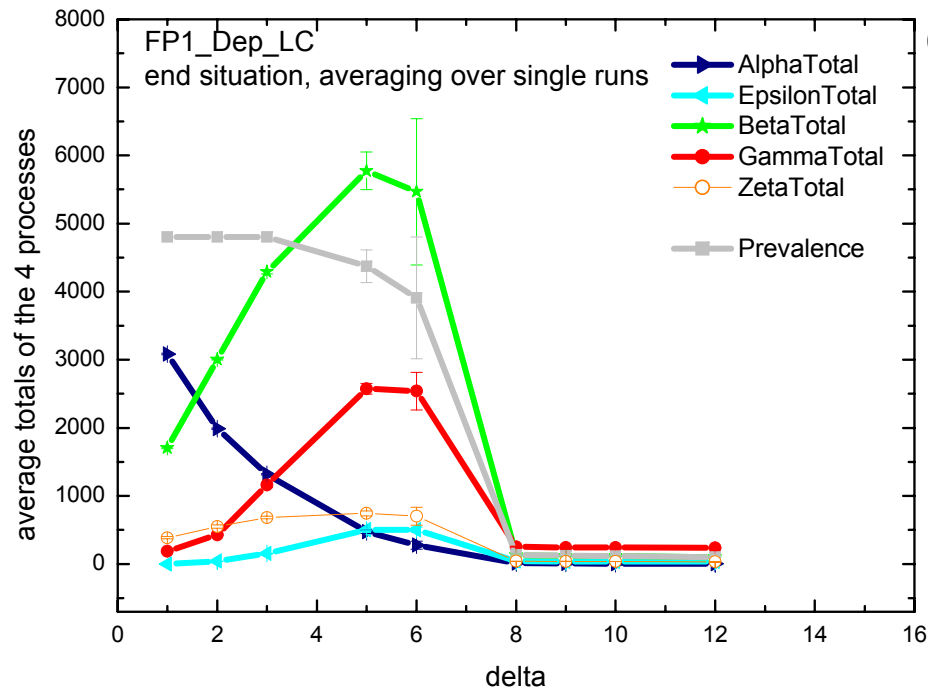
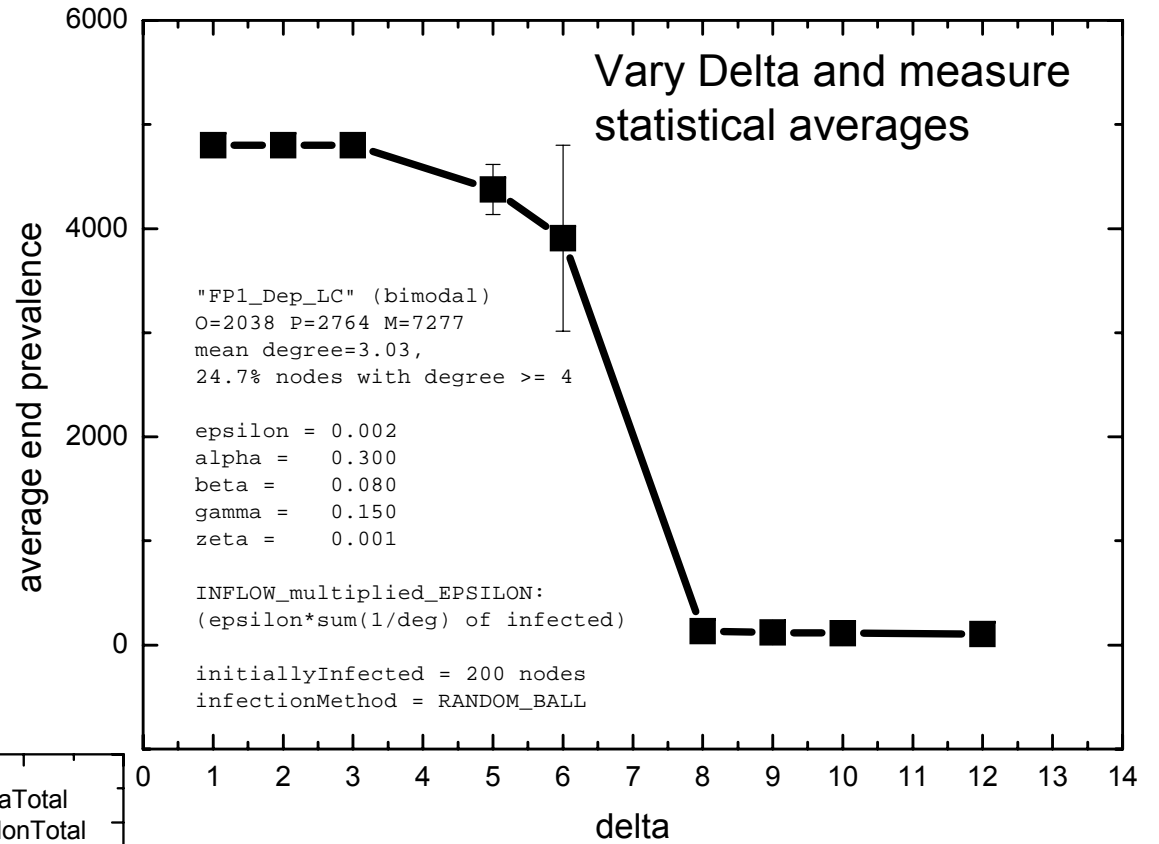
Many runs, averages of **end** results



# FP1

## Variation of delta

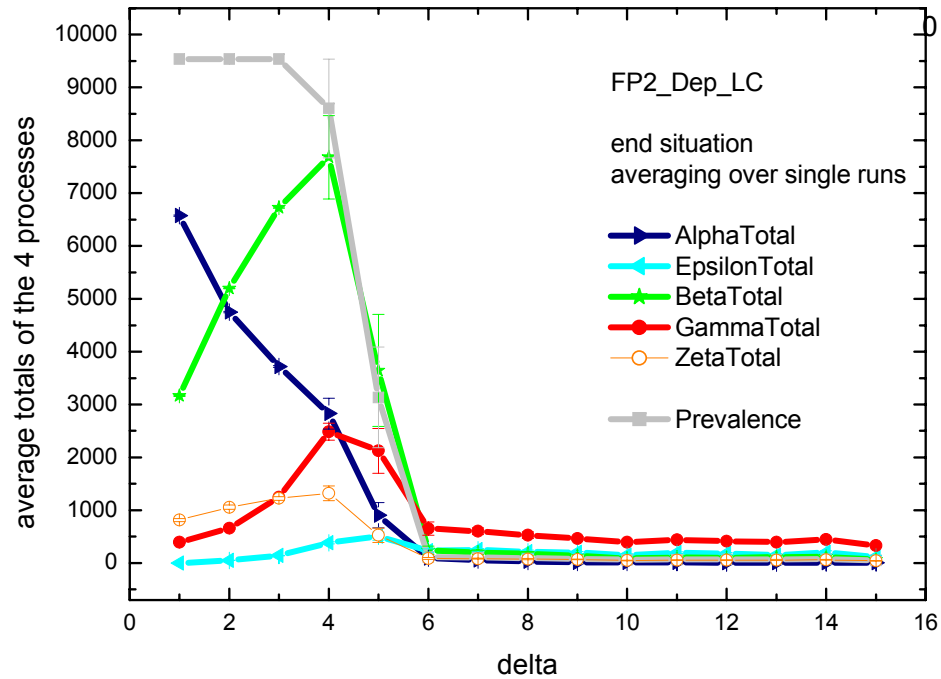
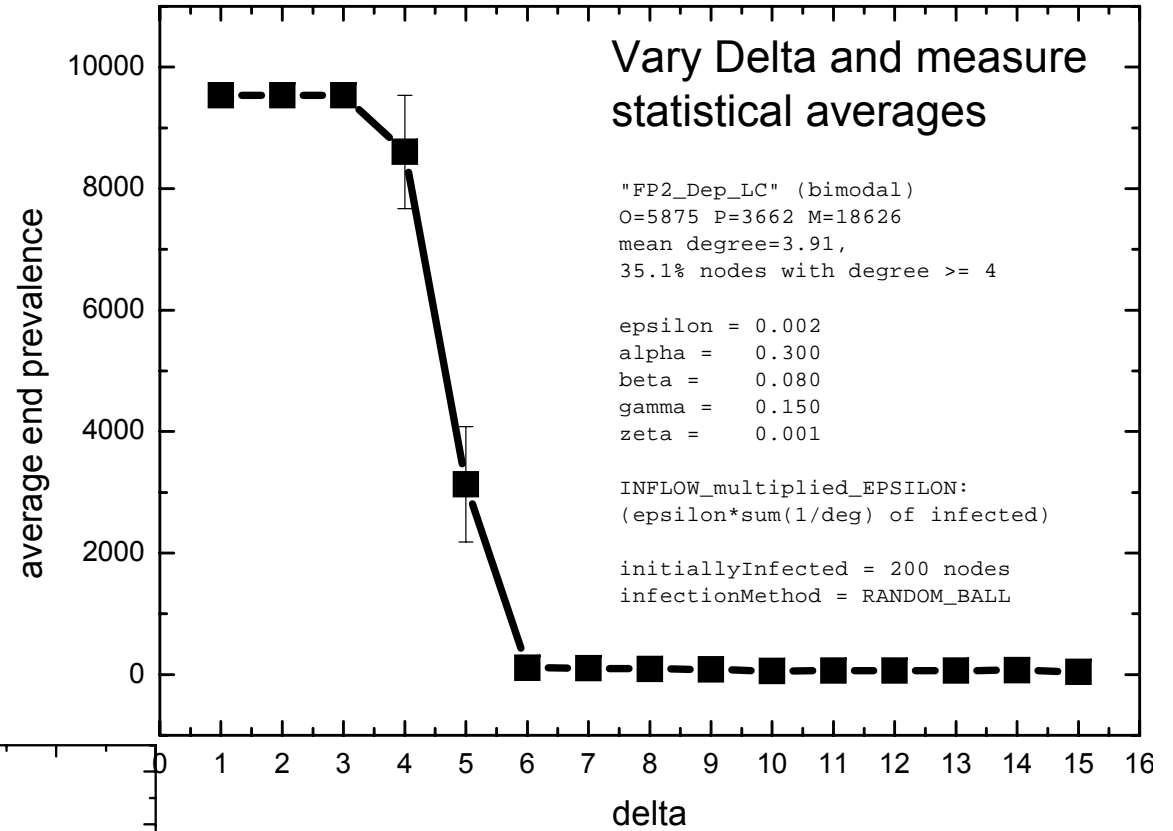
Many runs,  
averages of **end** results



# FP2

## Variation of delta

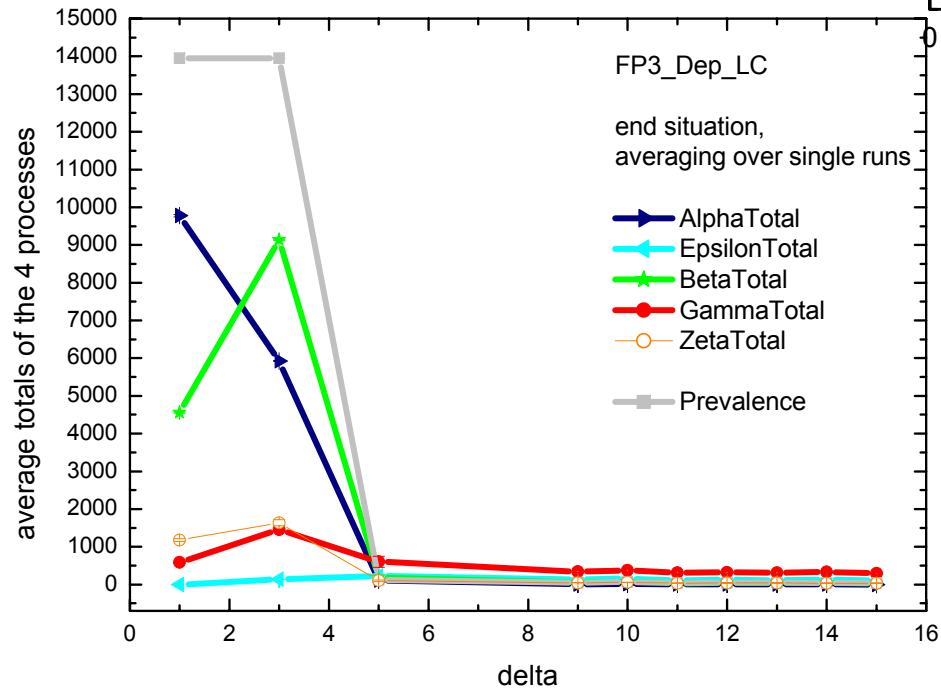
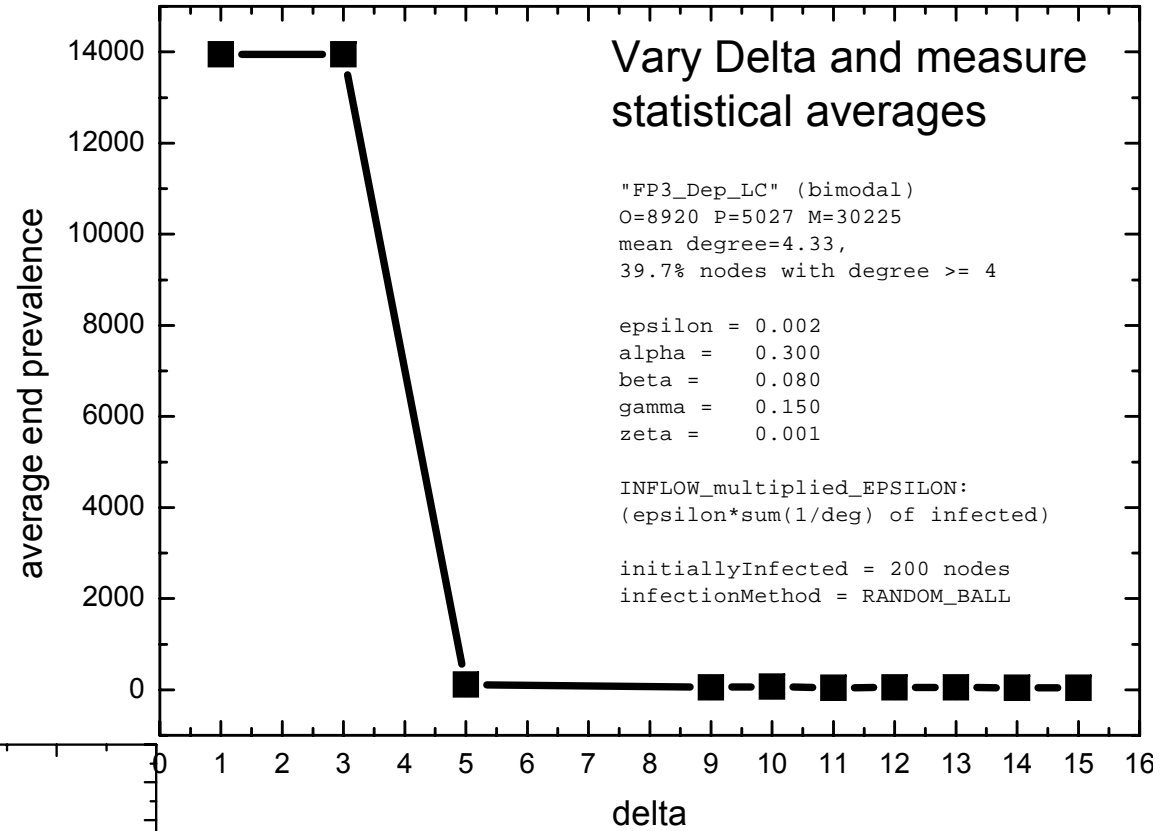
Many runs,  
averages of **end results**



# FP3

Variation of delta

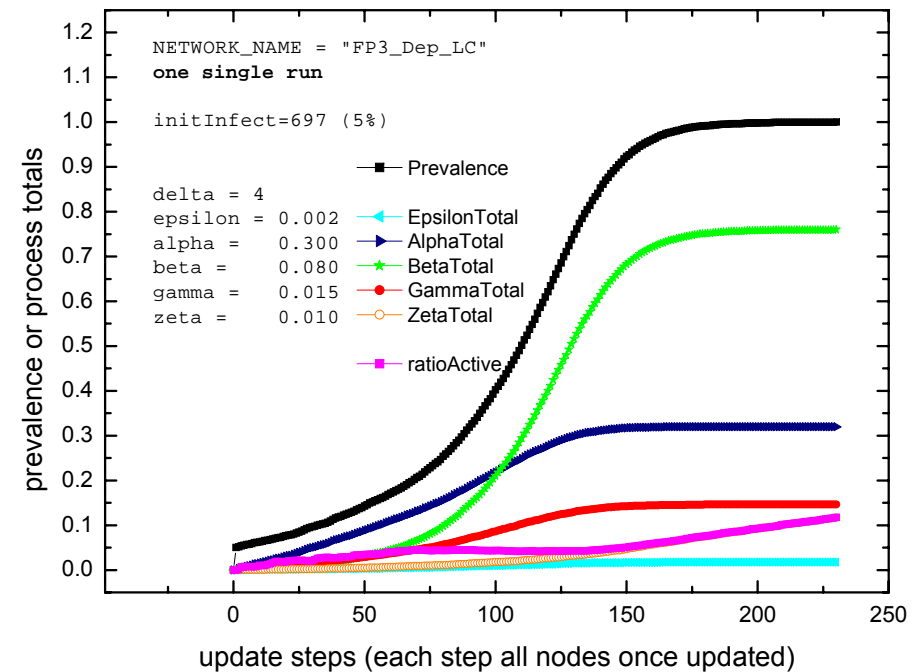
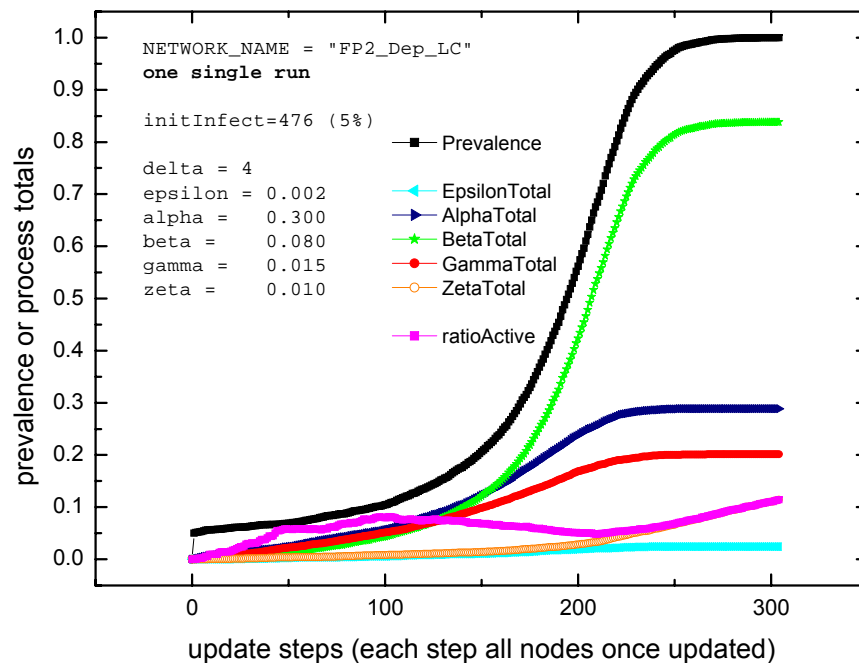
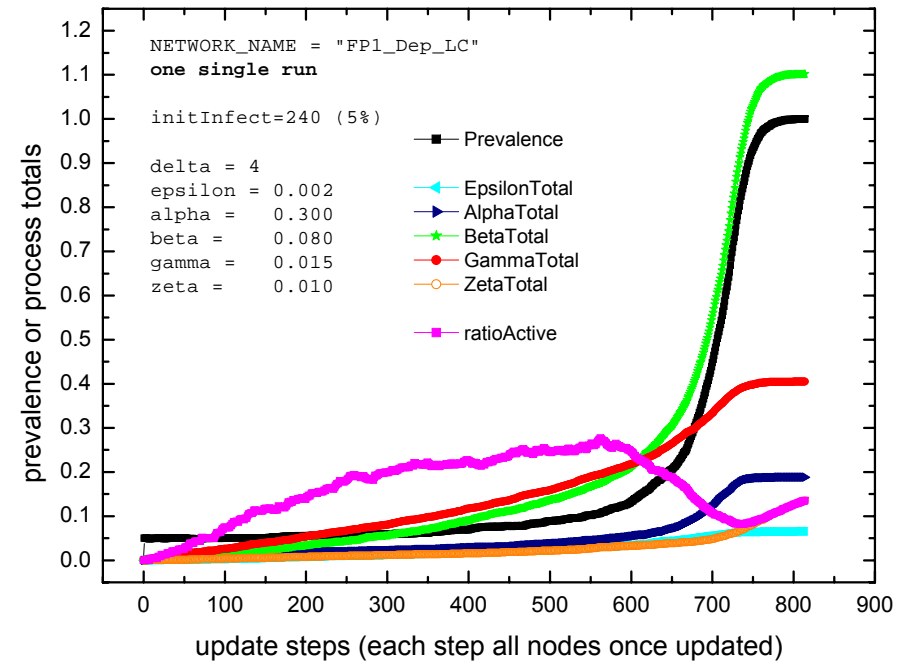
Many runs,  
averages of **end results**



Testweise nicht  
200 Anfangsinfizierte,  
sondern

5% Anfangsinfektion

240 Knoten, 476 Knoten, 697 Knoten  
je nach Gesamtsystemgröße





# Please...

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